Beta-Minus Emitters Dose Point Kernel Estimation Model Comprising Different Tissues for Nuclear Medicine Dosimetry Applications

Pedro Pérez[1,2,*], Federico Geser[1], Ignacio Scarinci[2], Francisco Malano[1,2] and Mauro Valente[1,2,3,*]

[1]*Instituto de Física Enrique Gaviola, FaMAF-UNC, CONICET, Córdoba, Argentina*

[2]*FaMAF-UNC, Córdoba, Argentina*

[3]*Centro de Física e Ingeniería en Medicina, Universidad de La Frontera, Temuco, Chile*

Abstract: The use of β-emitters for therapy purposes is one of the most extended procedures for tumor treatments in nuclear medicine practices over the last years. The constantly increasing dose delivery to healthy tissues in this practices, due to their high linear energy transfer and their radiobiological characteristics, might lead to complications in radiosensitive organs/tissues. Research efforts should be conducted to the development of tools and methods devoted to perform precise dosimetric calculations to deal with this issue and assess accurately dosimetric estimations on patients treated regions.

When performing dosimetry at organ level it is usual to assume some approximations on calculations, like uniformity in activity distribution within source regions, homogeneous media distribution for patient treated regions and uniform delivered dose on target organs. In this work, a formula to obtain Dose Point Kernel for different biological media is presented. Results are collated with Monte Carlo simulations suggesting a behavior that can be splitted in three groups, in accordance to their differences against the stochastic estimations: a) skin, blood and brain present differences within the 5% in comparison with the reference data; b) skeletal muscle, soft tissue, striated muscle and adipose tissue have differences lower than 20%; and c) compact bone, cortical bone and lung tissue differences are found above 50%.

This introduction of a medium-specific Dose Point Kernel calculation method could potentially lead to future improvements on dosimetric systems, limiting for now this model to tissues with effective atomic number closed to liquid water.

Keywords: Dose point kernel, Radioisotope, Monte carlo simulation, Theoretical models, Targeted radionuclide therapy.

1. INTRODUCTION

There are several investigations devoted to Targeted Radionuclide Therapy (TRT) improvements in nuclear medicine procedures [1-3]. TRT uses specific amounts of radiopharmaceuticals aiming at leaving precise radioactivity concentrations inside and around tumor tissues in order to eliminate diseased cells. The metabolic relation between patient and carrier presents non uniform activity distributions within body tissues and organs, while in TRT dosimetric methods almost all current protocols and procedures assume uniform distribution inside source organs/regions. An accurate assessment of dose deposition on target and healthy tissues in TRT procedures is still a non-completely solved issue in routinely clinics.

TRT dose distribution assessment can be carried out either by numerical or analytical methods. Numerically, it can be inferred through Monte Carlo (MC) simulations [4, 5], and analytically it can be derived from S values tables [6, 7] or by the Dose Point Kernels (DPK) convolution [8, 9] taking into account data from media and radionuclides involved. When performing dosimetry at organ level, it is customary to introduce some approximations like assuming uniformity in the activity distribution within the source regions and/or homogeneous tissue distribution on treated patient region. Similarly, the absorbed dose in target organs is represented as being uniform within the corresponding volume. At organ level dosimetry, source region uniformly activity distribution assumption results in a uniform deposited dose within both target and healthy tissues. Whereas that at voxel level dosimetry S values tables have been recently calculated for several radionuclides commonly used in nuclear medicine procedures and DPK convolution provides a powerful tool when the involved media can be assumed homogeneous.

*Address correspondence to this author at the Av. Haya de la Torre s/n. Ciudad Universitaria. Córdoba, Argentina.CP: X5000HUA;
Tel: +54 0351 433-4050 ext. 102; Fax: +54 0351 433-4054;
E-mail: pedro.perez@unc.edu.ar and valente@famaf.unc.edu.ar

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Nowadays, almost all used calculation tools in TRT are performed by a simplification of the Boltzmann radiation transport equation (RTE), taking into account infinite homogeneous media and then using the linearity principle to account for different dose contributions. On the other hand, MC simulations provide an accuracy in particle transport and energy delivery computation that cannot be achieved by analytical calculations. At present, new facilities allow high data processing that renders MC simulations an important tool to consider in the near future. The use of this tool to perform numerical calculations on complex systems allows to estimate accurate semi-analytical results [10].

The main goal of this work is to propose a model dedicated to analytical DPK assessment for dosimetric applications comprising different radionuclides of interest in nuclear medicine aiming at avoiding the high computational costs of performing them through MC simulations, or the intrinsic uncertainties of S values calculated on standard virtual phantoms. Thus, this model needs to be capable of estimating DPK curves for media of interest in nuclear medicine procedures, like different human tissues, by means of pre-calculated and known DPK curves from a pattern medium, *i.e*. water.

Some approaches have been carried out for soft tissues in terms of the RTE and it was found a simple formula to approximate several DPK radial distributions of interest from a known reference material. Liquid water was chosen as pattern because of its extended use for different purposes and the ease in finding it in published data. MC simulations have been run to obtain DPKs for comparing them with the estimations of the proposed approaching model. Finally, it was performed a comparison among simple applications on an arbitrary virtual phantom for the three analyzed calculation methods: MC simulation, traditional DPK convolution, and the DPK proposed model convolution.

The most important aspect of making it possible, considering different media in electrons emissions, is their short range in such media at energies of interest for radiopharmaceutical applications. In TRT procedures, the possibility of considering the specific source organ tissue for dosimetric calculations results in a powerful tool to improve the accuracy of computing dose distributions at a sub-organ and voxel level.

2. METHODS

2.1. Background Theory

2.1.1. Dose Approach by Means of Stopping Power Function

The RTE in the Continuous Slowing Down Approximation (CSDA) is also known as the Lewis equation (1950) since it was presented as a formula to describe the electrons transport [11] and can be written as

$$\hat{\Omega}\cdot\vec{\nabla}\phi+\Sigma_t\phi=Y(E)+\int\Sigma_e(\hat{\Omega}\rightarrow\hat{\Omega'})\phi\,d\hat{\Omega'}+\frac{\partial(S(E)\phi)}{\partial E} \quad (1)$$

where Σ_t and Σ_e are the total and elastic scattering macroscopic cross sections of the medium, respectively. $Y(E)$ is the source density spectrum representing the number of electrons generated with energy between E and E + dE and $S(E)$ is the *stopping power* function. Thus, when integrating over all the solid angle it becomes

$$\vec{\nabla}\cdot\vec{\Phi}=Y(E)+\frac{\partial(S(E)\Phi)}{\partial E} \quad (2)$$

where $\vec{\Phi}(E,r)$ refers to the vectorial expression of fluence and $\Phi(E,r)$ is the scalar fluence. The above integration cancels out all scattering terms. Therefore, by multiplying equation (2) by kinetic energy E and integrating them again, it is obtained that

$$\vec{\nabla}\cdot\vec{\Psi}=\int E\frac{\partial(S\Phi)}{\partial E}+Y_E \quad (3)$$

where $\vec{\Psi}(r)$ is the corresponding vectorial energetic fluence. Finally, when integrating by parts the first right term, it results in

$$\vec{\nabla}\cdot\vec{\Psi}=\int S\Phi dE+Y_E \quad (4)$$

Furthermore, the dose can also be expressed by [12]

$$D=-\frac{1}{\rho}\vec{\nabla}\cdot\vec{\Psi}+\frac{1}{\rho}\frac{d(\overline{\Sigma Q})}{dV} \quad (5)$$

from equation (5) it is observed that $\Sigma Q=Y_E-Q_r$; where Y_E is the total kinetic energy of emitted radiation by medium mass unit from the source and Q_r is the net increment in rest energy by unit mass of the medium due to radiation. So, absorbed dose D turns out as

$$D=\int\Phi(E,r)\frac{S(E)}{\rho}dE \quad (6)$$

Now, the equation (6) expresses D by means of *stopping power* function, $S(E)$.

2.1.2. Scaled DPK

When considering DPK as a consequence of β particles radiation, it is customary to designate it as βDPK. Although this work considers only electron (β-minus) sources, the term DPK will be used. It can be considered a simplified model of a point source isotropically emitting monoenergetic electrons moving through a homogeneous medium and loosing energy slowly and continuously (CSDA conditions) according to $S(E)$ [8]. A scaled quantity of DPK, called sDPK or *F* in literature, will be defined, with the purpose of studying it over a wide energy range, as follows

$$F(r,E_0)=\frac{\delta E(r)/E_0}{\delta r/R_{CSDA}} \tag{7}$$

where r is the distance to the source, $\delta E(r)$ is the energy fraction delivered to the body within r and $r+\delta r$, E_0 is the initial kinetic energy of the electron and R_{CSDA} is the electron range in the CSDA for E_0. Besides, F can be described as a function of $S(E)$ as follows [11]

$$F(r,E_0)=\frac{R_{CSDA}\,S(E(r,R_{CSDA}))}{E_0}=\frac{S(E(r,R_{CSDA}))}{\langle S\rangle} \tag{8}$$

where the parameter $\langle S\rangle=\frac{E_0}{R_{CSDA}}$ has been introduced. In this sense, F can be interpreted as the ratio of $S(E)$ in a particular point on the electron track and its value along the whole traveled path.

2.1.3. Scaled DPK Approximation Modeling

It can be directly performed a generalization of expression (8) for the emission due to an arbitrary spectrum $P(E)$ over an energy range $[E_{\min},E_{\max}]$. Thus, this model can be applied for realistic radionuclides dosimetric calculations.

$$\int_{E_{\min}}^{E_{\max}}F(r,E_0)dE\approx\sum_j\frac{p_j S(E_j)E_j}{\langle S(E_j)\rangle} \tag{9}$$

where ρ_j and E_j are the corresponding weight factor and discrete energy for the spectrum $P(E)$.

Moreover, Bethe-Bloch $S(E)$ expression for charged particles at relativistic energies can be written as [13]

$$S(E)=4\pi\frac{e^4}{\mu_0}\left(\frac{z_p}{v_p}\right)^2N_A\frac{\rho Z}{AM_\mu}\left[In\left(\frac{2m_0v_p^2}{I}\right)-In\left(1-\frac{v_p^2}{c^2}\right)-\frac{v_p^2}{c^2}\right] \tag{10}$$

where e and m_0 represents electron charge and rest mass, respectively. z_p and v_p are the atomic number and velocity of the incident particle, N_A is the Avogadro's number and M_μ is the molar mass constant. Finally, ρ, Z, I and A represent mass density, effective atomic number, mean ionization energy and relative atomic mass of the absorber medium, respectively.

It can be inferred from expression (10) that for non-relativistic conditions the last two terms of the equation are closed to zero and might not be considered for a first approach. However, a rigorous analysis for TRT applications may take into account theses terms (*e.g.* for a 100keV kinetic energy on incident particle results in $v^2/c^2\approx 0.3$).

It is desirable to approximate an expression of an unknown F by means of an already known scaled DPK considering only a few parameters of the material of interest. This possibility would avoid the need of calculating F for every tissue and every radioisotope and, therefore, only a reduce quantity of F estimations would be necessary for each radioisotope. Then, F could be assessed from the known data of the reference material. From equation (8) it follows that

$$\frac{F_{med}}{F_{ref}}=\frac{S_{med}}{\langle S_{med}\rangle}\frac{\langle S_{ref}\rangle}{S_{ref}} \tag{11}$$

where subscripts *med* and *ref* refer to absorber and reference medium, respectively. Then, equation (11) can be rewritten as

$$F_{med}=\mathcal{T}_{med,ref}\cdot\mathcal{T}'_{med,ref}\cdot F_{ref} \tag{12}$$

where $\mathcal{T}'_{med,ref}=\langle S_{ref}\rangle/\langle S_{med}\rangle$ and $\mathcal{T}_{med,ref}$ can be expressed, according to Beth-Bloch expression (10), in terms of E as [14]

$$\mathcal{T}_{med,ref}=\frac{\rho_{med}}{\rho_{ref}}\frac{Z_{med}}{Z_{ref}}\frac{A_{ref}}{A_{med}}\left[\frac{In\left(\frac{\alpha(E)}{I_{med}}\right)-\delta(E)}{In\left(\frac{\alpha(E)}{I_{ref}}\right)-\delta(E)}\right]$$

$$=\rho_{ref}^{med}Z_{ref}^{med}A_{med}^{ref}\left|\frac{In\left(\frac{\alpha(E)}{I_{med}}\right)-\delta(E)}{In\left(\frac{\alpha(E)}{I_{ref}}\right)-\delta(E)}\right| \tag{13}$$

where notation a_i^j means $\frac{a_j}{a_i}$ and

$$\alpha(E) \equiv 2\frac{E^2 + 2Em_0c^2}{m_0c^2}$$

$$\delta(E) \equiv 1 - \frac{m_0c^2}{E + m_0c^2} \quad (14)$$

While energies involved in TRT procedures range from a few hundred eV to the order of MeV, the really important issue is that useful resolutions for dosimetric distribution estimations varies from 1 to 3 mm according to resolution achieved by commonly clinical machines, *i.e*. PET, CT. So, an analysis on $E \in$ (100eV, 1MeV) is enough for these purposes, especially considering the low values of R_{CSDA} for electrons traveling in water with low energies (*e.g*. $R_{CSDA} = 0.00025cm$ for 10 keV).

When considering radiation transport in TRT procedures, materials correspond to human tissues that present mean ionization energy similar to water's, I_{wat}. Then, water is established as the reference medium and it will be used for F calculations by means of the well-known and extensively published water F curve on different conditions. Considering electron initial energies between 100eV and 1MeV then $\alpha(E)$ ranges from 400eV to 7.91MeV. Besides, for studied tissues with ionization energies from 72.3 to 106.4 eV then $\alpha(E)/I$ ranges from 109.46keV to 4eV and finally $ln(\alpha(E)/I) \in$ (1.5,11.6). Moreover, the results of valuating $\delta(E)$ for that energy range are between 0.00039 and 0.88563. Thus, the influence of $\delta(E)$ is at most 8.45% and equation (13) can be approximated, in first approach, to

$$\mathcal{T}_{med,wat} \approx \rho_{wat}^{med} Z_{wat}^{med} A_{med}^{wat} \left[\frac{In\left(\frac{\alpha(E)}{I_{med}}\right)}{In\left(\frac{\alpha(E)}{I_{ref}}\right)} \right] \quad (15)$$

Lastly, from (12) and (15) it is found an expression to obtain scaled DPK for electron's energies and materials of interest in nuclear medicine from a previously known curve for water as

$$F^{med} \approx \rho_{wat}^{med} Z_{wat}^{med} A_{med}^{wat} R_{wat}^{med} I_{med}^{wat} F^{wat}$$

$$F^{med} \approx \gamma(\rho, Z, A, R, I) F^{wat} \quad (16)$$

where $R = R_{CSDA}$ and $\gamma(\rho, Z, A, R, I)$ is a conversion factor associated with the proposed approach. In this last, second, approximation, the ratio of the ultimate term of expression (15) was taken as the ratio of its arguments.

2.2. Scaled DPK Calculations

Radial dose distributions were calculated around point monoenergetic electron sources within several infinite media by means of MC simulations. F was calculated for each energy and material performing an extensive database of DPKs of interest.

Each DPK calculation has been simulated within concentric spheres and the delivered energy was tallied in concentric shells of $0.025 \cdot R_{CSDA}$ thicknesses up to $1.5 \cdot R_{CSDA}$. R_{CSDA} values have been calculated following two methods: through PENELOPE MC general-purposes code libraries and and also by means of physical properties obtained from NIST (National Institute of Standards and Technology, USA) data tables through ESTAR online available software. In all cases the β^- emission in biological tissues was studied and the corresponding *F* was calculated.

A full stochastic calculation system was developed based on physics' package of the PENELOPE v.2008 [15] MC main code capable of voxel level radiation transport and dosimetry. Also, it can perform dosimetry over definite geometries and it is able to estimate delivered energy to user-defined bodies due to primary particles such as photons, electrons and positrons. Moreover, the developed system provides tools able to discriminate dose contributions due to primary and scattered particles.

User interfaces, data analysis and visualization were developed both on a MATLAB platform and in scientific Python language, and simulations have been run in laboratory computers with i7 cores.

2.3. Assessment of Uncertainties Involved in the Proposed Approach

The approximations included in equations (13) and (15) may involve uncertainties because $\delta(E)$ was neglected. This approach implies differences (Dif) that can be evaluated by

$$Dif = \left| \frac{\frac{In\left(\frac{\alpha(E)}{I_{med}}\right) - \delta(E)}{In\left(\frac{\alpha(E)}{I_{ref}}\right) - \delta(E)} - \frac{In\left(\frac{\alpha(E)}{I_{med}}\right)}{In\left(\frac{\alpha(E)}{I_{ref}}\right)}}{\frac{In\left(\frac{\alpha(E)}{I_{med}}\right) - \delta(E)}{In\left(\frac{\alpha(E)}{I_{ref}}\right) - \delta(E)}} \right| \quad (17)$$

On the other hand, the second approximation consists on taking the ratio of the last term in equation (15) as the ratio of its arguments. Thus, this approach involves uncertainties that need to be accurately estimated too, following the same procedure that was shown in equation (17).

2.4. Calculation of Kernels for Tissues

According to the proposed method for the calculation of F it is necessary to introduce some specific physical parameters. At least ρ, Z, I and A from both the medium from which F needs to be assessed and the chosen reference medium. As usually, liquid water was considered as reference material. Also, it is necessary to obtain the electron ranges for the corresponding media and energies and a well-known curve for F_{wat}.

The considered biological tissues are defined in accordance to ICRP (brain, cortical bone, skeletal muscle, skin, lung, blood, adipose tissue and soft tissue) and ICRU (striated muscle and compact bone) formalisms and PENELOPE v.2008 MC code databases (liquid water).

The effective atomic number has been calculated by the formula [16]

$$Z^{2.94} = \sum_{i=1}^{n} a_i Z_i^{2.94} \quad (18)$$

where a_i is the fraction of the total number of electrons associated with Z_i atomic number in the tissue molecule. While relative atomic mass A has been calculated by [17]

$$A = \sum_{i=1}^{n} a_i A_i \quad (19)$$

where A_i is de atomic mass of each element present in the tissue.

Material mass density and corresponding mean ionization potential were obtained from PENELOPE database.

2.5. Performance Evaluation and Applications

Preliminary tests were carried out using a virtual phantom containing four spheres with different uniform activities emulating organs/tissues. Calculations considered emissions from homogeneous distribution of electron sources inside the spheres.

A code written in Python computational language was developed to perform dosimetry through F convolution, and MC calculations for 3D dosimetry in the definite phantom have been run on the FLUKA MC code [18].

The calculations for F curves have been computed through MC simulation and the proposed model using water as reference medium. These methods were compared and analyzed along with their advantages and disadvantages.

3. RESULTS

3.1. Preliminary Analysis of the Implemented Approach

Aiming at evaluating correctly the performance of equation (16) for the seven biological tissues of interest, it is necessary to quantify uncertainties involved in previous approaches, comprising this media. For all of this evaluations it is defined water as the reference material. Thus, first approximation is analyzed following equation (17) and it was found that the larger differences against the calculation of the full expression (10) in this approach are always less than 2×10^{-4} whereas the lower differences, around 4×10^{-7} correspond to soft tissue and blood. Figure **1** reports this calculated maximum and minimum differences.

Then, it is necessary to evaluate the second approach where the ratio of the ultimate term of expression (15) was taken as the ratio of its arguments. An exhaustive analysis of these possible discrepancies was made. Then, results in the minimum and maximum differences are summarized in Figure **2**. It is verified that differences due to the implementation of the method are always less than 0.0341 for soft tissue and the minimum value is found for blood (0.0018).

Differences	Brain	Skin	Blood	Lung	Soft Tissue	Striat. Muscle	Skel. Muscle
Maximum ($\times10^{-6}$)	3.801	5.149	0.445	0.668	6.051	0.669	0.668
Minimum ($\times10^{-4}$)	1.664	2.259	0.194	0.290	2.657	0.291	0.290

Figure 1: Discrepancies for the first approximation in the proposed model.

Differences	Brain	Skin	Blood	Lung	Soft Tissue	Striat. Muscle	Skel. Muscle
Maximum	0.0156	0.0212	0.0018	0.0027	0.0251	0.0027	0.0027
Minimum	0.0212	0.0289	0.0024	0.0036	0.0341	0.0037	0.0036

Figure 2: Discrepancies for the second approximation in the proposed model.

3.2. Calculation of DPK

The developed code was compared against results obtained by Botta *et al*. [19] with the purpose of establishing its accuracy and validation. This analysis shows a good agreement between results of both calculations, always within the statistic deviations. So, results obtained by Botta *et al*. are considered undistinguishable as regards the ones obtained using the developed system.

A percentage variation parameter $\Delta F\%$ has been estimated according to the equation $\Delta F\% = (|(F_{Botta} - F_{Calculated})| / F_{Calculated}) \times 100$. It shows higher differences at distances above r / R_{CSDA}, just on the region where the delivered energy is closed to 0 eV, so the main reason for the differences is due to statistical fluctuations that become relatively important when absolute magnitudes of calculated quantities goes to zero, like energy in this case. But, for example, 1MeV electrons' differences give $\Delta F\%$ lower than 0.7 at distances within the R_{CSDA}.

Moreover, Figure **3** shows some results obtained for blood and soft tissues corresponding to monoenergetic electron point sources of 200 keV and 1 MeV initial kinetic energies, respectively. In these figures, *F* represents the MC calculated scaled DPK using the developed code, while F_{prim} and F_{scat} refer to primary and scattering contributions to the total *F*.

Besides, considering only primary particles contributions, it results that there is no considerable variation on calculus uncertainties in energy variable. Figure **4** shows a case of in-brain simulations for 20 keV and 2 MeV and variation within the R_{CSDA} and in the peak zone never reaches 2%. 20 keV and 2 MeV correspond to the minimum and maximum studied initial electron kinetic energies, respectively.

A detailed results analysis suggest a behavior that might be analyzed in three well-defined groups. The first one, where the model can be considered undistinguishable in comparison with MC calculations (differences < 5%): skin, blood and brain belong to this first group. The second one where the uncertainties have to be considered prior to evaluating the convenience of making dosimetry using the model (differences < 20%): skeletal muscle, soft tissue, striated muscle and adipose tissue are part of this

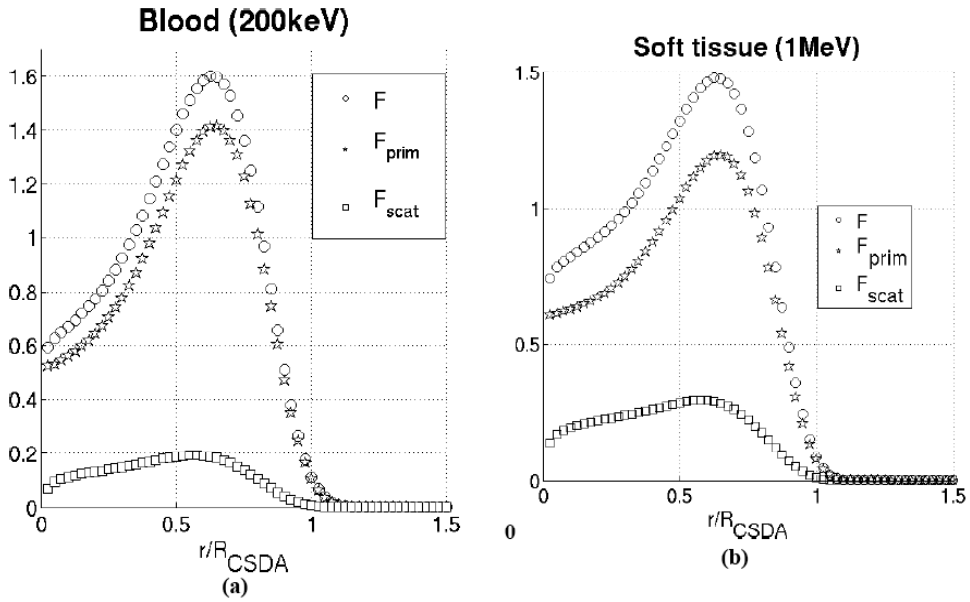
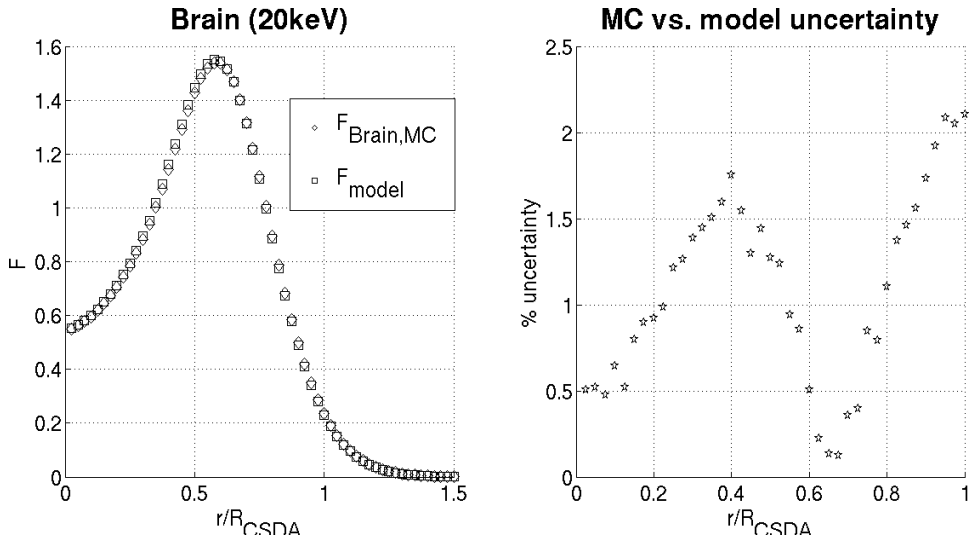
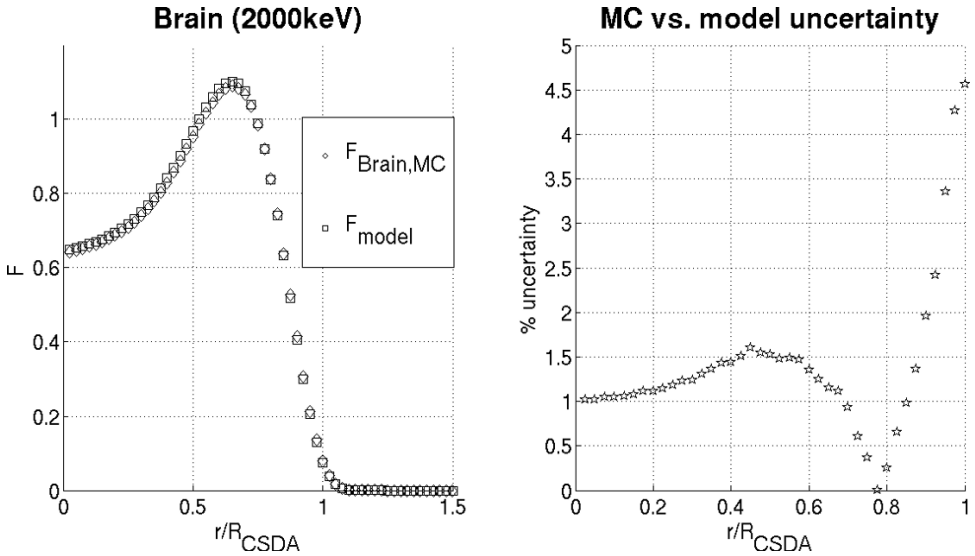


Figure 3: Results for *F* in blood (200 keV) and soft tissue (1 MeV) discriminating primary and scattering contributions to energy deposition.



(a)



(b)

Figure 4: In-brain comparisons between MC and proposed model calculations from minimum (**a**) and maximum (**b**) initial electrons energies studied.

group. And the third one, where the model, at least taking into account water as a reference medium, cannot be applied in the terms shown in this work (differences > 50%): compact and cortical bone, and lung tissue belong to this group.

Finally, when the performance of the model is analyzed by using the total F_{wat} MC assessed for calculations, the results are similar to the previous ones, but with increasing uncertainties. The first group has maximum uncertainties of 15% for initial source electron energies of 200keV but reaching 25% in some zones for 1MeV sources. The second group shows better results for the model than primary contribution calculations. In this case, the larger differences are found at 1MeV in adipose and soft tissues reaching 20%, but in generally closed to 10%; while the third group is still not recommendable but in cases of compact and cortical bone the model presents differences lower than 50%.

3.3. Applications of the Proposed Method

Simulations were run on a virtual phantom considering four different spheres with different homogeneous activity in each one. A simplified version of typical applications was studied over this virtual phantom to evaluate the model performance against both standard DPK convolution and MC simulation methods.

The model has been evaluated considering primary and total contributions to dose due to activity distribution in phantom. Primary contribution, as it was mentioned above, considers only the dose due to primary particles from the source; while total contribution considers all the different contributions to the dose, even secondary particles and scattering photons.

Figure **5**(**a**) reports results for brain tissue in phantom comparing standard DPK convolution against the proposed model taking into account only primary particles contributions to delivered dose. Figure **5**(**b**) compares the same issue but considering differences between standard DPK convolutions against a MC simulation run on FLUKA MC code. Then, Figure **5**(**c**) shows results of differences between the proposed model and MC simulation.

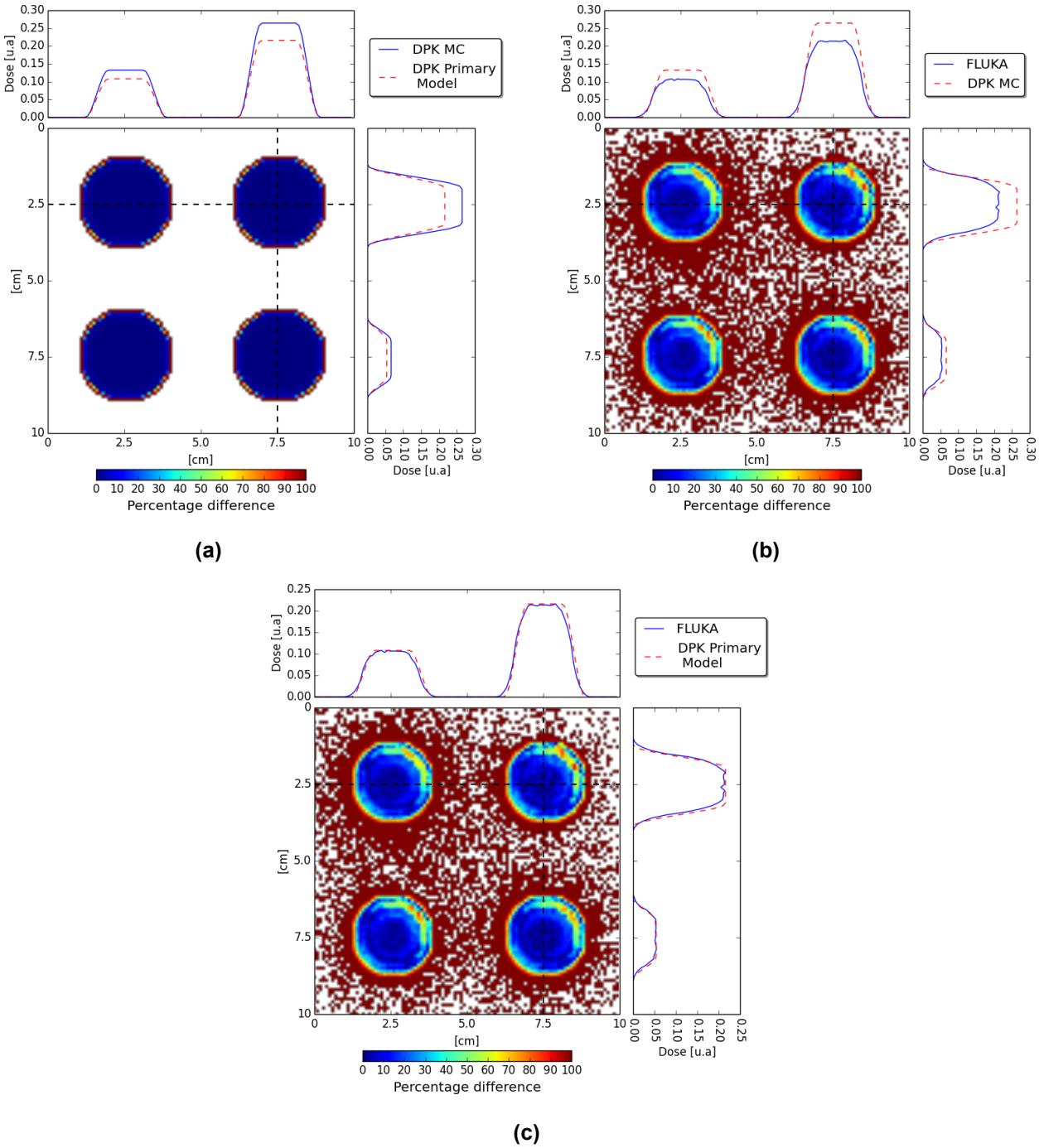


Figure 5: Differences in dose distribution on brain comparing the proposed method considering only primary contribution with standard DPK convolution (**a**), DPK standard convolution versus MC (**b**) and proposed method versus MC (**c**).

Finally, Figure **6** shows results of differences between proposed model and the MC simulation run on FLUKA MC codeon striated muscle taking into account the whole dose (all of its contributions).

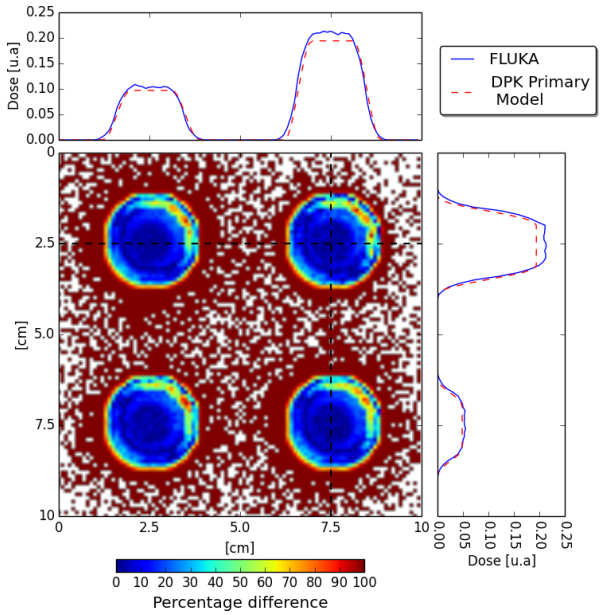


Figure 6: Differences in dose distribution on striated muscle comparing the proposed method considering total contributions with MC simulation using FLUKA.

4. DISCUSSION

In view of the results reported for validation and the application of the model to dose distribution assessment by DPK calculations, it must be highlighted that it was found a promising overall performance. The approach is consistent with the formalism of the main Boltzmann radiation transport equation incorporating the CSDA. It was proposed an approach for the scaled DPK as a function of the Stopping Power. Then, from Bethe-Bloch formulation it was possible to obtain the scaled DPK, *F*, for electron sources in definite media by means of a previously well-known reference one.

Looking for materials of interests in TRT procedures, there were performed *F* for β^- monoenergetic sources at several energies through both MC method and the proposed model. Also, it was developed a specific code aimed at calculating deposited dose taking into account discriminated contributions both from primary and secondary/scattered particles. An exhaustive analysis on the developed code shows that the results are undistinguishable from those presented in bibliography.

Besides, a study of the proposed approaches for F calculation in the framework called "first approximation", without consideration of $\delta(E)$ of expression (13), shows excessively large uncertainties. But, the further analysis based on the "second approximation" shows analytical uncertainties always <3% for energies and materials of interest in nuclear medicine practices.

Finally, it was possible to compute F for a wide variety of cases using the proposed method taking water as the reference pattern. Moreover, it was shown how to implement the model in terms of primary and total contributions to the dose deposition. This kind of discrimination allows to evaluate the performance in both cases. A rigorous analysis establishes that secondary particles might exhibit, in average, minor increment in Linear Energy Transfer (LET) with respect to primary particles of higher energies. Thereby, this capacity of the calculation model may constitute a valuable tool to perform LET-weighted dosimetry and a more realistic evaluation case-by-case in future tools developments.

It was found that, when considering the second group of materials, the model attains the best performance calculating with total (primary and scattering) dose; whereas the deviations of results obtained for the third group may constitute significant limitations for the direct implementation in nuclear medicine dosimetry. Some approaches and further modifications of the model aimed to incorporate effective traveled path according to mass density, currently under investigation, may provide improved performance. For now, it is not suitable to implement the method when involved materials like cortical bone, compact bone or lung tissue.

As final remarks, it must be emphasized that this work presents a novel method that appears as a promising tool for dosimetric assessment by means of DPK convolution for at least six different biological tissues commonly involved in nuclear medicine dosimetry: brain, blood, skin, skeletal muscle, soft tissue, striated muscle and adipose tissue. Tissues with mass density and ionization energy away from water values are still under investigation and the proposed method in the current version should not be recommended in these cases.

CONCLUSIONS

It was possible to achieve an expression to evaluate the absorbed dose in different materials by means of

the Stopping Power function through analytical calculations based on fundamentals of radiation transport physics. The two proposed approaches for the calculation of F in terms of a reference F provided consistent results. Also, results summarized in Figures **1** and **2** support that the theoretical background of the developed system can achieve a good performance for dosimetric purposes.

The proposed model was satisfactory validated with published data [19] and MC simulations. In addition, preliminary tests performed on simplified, but somehow realistic, phantoms demonstrated the reliability of the proposed method also pointing out specific characteristics of its performance in different situations when compared with standard DPK and MC calculations. Actually, results show that the model attains good performance even in cases presenting in homogeneities thus converting the model into a more powerful tool to improve dosimetry in TRT.

Finally, it was found that the method is ready to be implemented in dosimetric calculations involving brain, blood and skin, otherwise the use of the method for skeletal muscle, soft tissue, striated muscle and adipose tissue must be taken carefully considering the important uncertainties involved, and by last it was established that the method is not suitable yet for applications were dosimetry needs to be performed on compact and cortical bone and lung tissue.

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