Integral Type Contraction in Dislocated Metric Spaces

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Abstract
In this paper, by using the notion of compatibility, weak compatibility, occasionally weak compatibility and commutativity, we establish some common fixed point theorems for six mappings satisfying integral type contractive conditions in complete dislocated metric spaces. Our work improves and extends some earlier results in the literature.

Keywords: Dislocated Metric Space, Compatibility, Weak Compatibility, Common Fixed Point, Cauchy Sequence, Commuting Mapping, occasionally weak compatibility.

1.Introduction

The notion of metric space, introduced by Frechet [6] in 1906, is one of the useful topic not only in mathematics but also in several quantitative sciences. Due to its importance and application potential, this notion has been extended, improved and generalized in many ways. An incomplete list of such attempts are following: symmetric space, b-metric space, partial metric space, quasi-metric space, fuzzy metric space, dislocated metric space, dislocated quasi-metric space, right and left dislocated metric spaces etc.

In 1922, Banach proved a fixed point theorem for contraction mapping ina complete metric space. Banach contraction theorem is one of the pivotal results of functional analysis. It has many applications in various fields of mathematics such as differential equations, integral equations etc. After Banach contraction theorem number of fixed point theorems have been es- tablished by various authors and they made different generalizations of this theorem.

The concept of dislocated metric (*d*-metric) was introduced by Hitzler and Seda in [15,16] which is very useful in Logic Programming Semantics. With the passage of time many papers have been published concerning fixed point and common fixed point theorems satisfying certain contractive conditions in dislocated metric space (see [16]–[24]).

Branciari [2] introduced the notion of contraction of integral type and proved first fixed point theorem for this class of mapping. Further results on this class of mappings were obtained by Rhoades [24], Aliouche[3], Djoudi and Merghadi [5] and many others.

Sessa [27], initiated the tradition of improving commutativity conditions in metrical common fixed point theorems. While doing so Sessa [27] introduced the notion of weak commutativity. Motivated by Sessa [26], Jungck [9] defined the concept of compatibility of two mappings, which includes weakly commuting mappings as a proper subclass. Jungck and Rhoades [13] introduced the notion of weakly compatible (coincidentally commuting) mappings and showed that compatible mappings are weakly compatible but not conversely. Many interesting fixed point theorems for weakly compatible maps satisfying contractive type conditions have been obtained by various authors.In this paper, we have established some common fixed point results of integral type contractive conditions using the concept of compatibility, weak compatibility and commutativity in complete dislocated metric (*d-metric*) spaces. Our obtained results generalize some well known results of the literature.

2. Preliminary Notes

We begin by recalling some basic concepts of the theory of dislocated metric (*d-metric*) spaces.

Throughout this work R^+ represent the set of non-negative real numbers. Now, we collect some known definitions and results from the literature which are helpful in the proof of our results.

Definition.2.1: Let X be a nonempty set. Suppose that a mapping $d: X \times X \to R^+$ satisfies:
(i) $\int_0^{d(x,y)} \emptyset(t)dt = 0 \;\forall\; x, y \in X$
(ii) $\int_0^{d(x,y)} \emptyset(t)dt = \int_0^{d(y,x)} \emptyset(t)dt = 0 \Rightarrow x = y$

(iii) $\int_0^{d(x,y)} \emptyset(t)dt = \int_0^{d(y,x)} \emptyset(t)dt$

(iv) $\int_0^{d(x,y)} \emptyset(t)dt \leq \int_0^{d(x,z)} \emptyset(t)dt + \int_0^{d(z,y)} \emptyset(t)dt$ for all $x, y, z \in X$

Then *d* is called a metric on *X* and (*X*, *d*) is called a metric space. If *d* satisfies the conditions from(ii)-(iv),,then *d* is said to be dislocated metric (OR) shortly (*d*-metric) on *X* and the pair (*X, d*) is called dislocated metric space. If *d* satisfies only (ii) and (iv), then *d* is called dislocated quasi-metric (OR) shortly (*dq*-metric) on *X* and the pair (*X, d*) is called dislocated quasi-metric space.

Where $\emptyset : R^+ \to R^+$is a Lebesque integrable mapping which is summable on each compact subset of R^+, non-negative and such that for any $s > 0$, $\int_0^s \emptyset(t)dt > 0$

Note. The above definition change to usual definition of metric space if $\emptyset(t) = I$

It is clear that every metric space is dislocated metric and dislocated quasi metric space but the converse is not true. Also every dislocated metric space is dislocated quasi-metric space but the converse is not necessarily true.

Definition 2.2 A sequence $\{x_n\}$ in a *d*-metric space (X, d) is called a Cauchy sequence if for given $\epsilon > 0$, there exists $n_0 \in N$ such that for all $m. n \geq n_0$,we have $d(x_m, x_n) < \epsilon$.

The following simple but important results can be seen in [11].

Definition 2.3 A sequence in *d*-metric space converges if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.

Definition 2.4 A d-metric space (X, d) is called complete if every Cauchy sequence is convergent.

Definition 2.5 Let (X, d) be a d-metric space. A map $T: X \to X$ is called contraction if there exists a number λ with $0 \leq \lambda < 1$ such that $d(Tx, Ty) \leq \lambda d(x, y)$.

Branciari [2] proved the following theorem in metric spaces.

Theorem 2.1. Let (X, d) be a complete dislocated metric space for α ∈ (0, 1). Let T : X →X be a mapping such that for all$x, y \in X$ satisfying

$\int_0^{d(Tx,Ty)} \emptyset(t)dt \leq \alpha \int_0^{d(x,y)} \emptyset(t)dt,$

where $\emptyset : R^+ \to R^+$is a Lebesque integrable mapping which is summable on each compact subset of R^+, non-negative and such that for any $s > 0$, $\int_0^s \emptyset(t)dt > 0$.

Definition 2.6 Let *A* and *S* be two self mappings on a set *X*. Mappings *A* and *S* are said to be commuting if $ASx = SAx \quad \forall x \in X$.

Definition 2.7. Let S and T be mappings of a dislocated metric space (X, d) into itself. Then (S, T) is said to be **weakly commuting** pair if

$d(STx, TSx) \leq d(Tx, Sx)$ for all $x \in X$.

Obviously a commuting pair is weakly commuting but its converse need not be true as is evident from the following example.

Example 2.1. Consider the set X = [0, 1] with the usual metric. Let $Sx = \frac{x}{2}$ and $Tx = \frac{x}{2+x}$ for every $x \in X$. Then for all $x \in X$

$STx = \frac{x}{4+2x}, \; TSx = \frac{x}{4+x}$.

Hence $ST \neq TS$. Thus S and T do not commute.

Again

$$d(STx, TSx) = \left|\frac{x}{4+2x} - \frac{x}{4+x}\right| = \frac{x^2}{(4+x)(4+2x)}$$
$$\leq \frac{x^2}{(4+2x)} = \frac{x}{2} - \frac{x}{2+x} = d(Sx, Tx),$$

and so, S and T commute weakly.

Obviously, the class of weakly commuting is wider and includes commuting mappings as subclass.

Definition 2.8. Two self mappings *S* and *T* of a complete dislocated metric space (*X*, *d*) are compatible if and only if $\lim_{n\to\infty} \int_0^{d(STx_n,TSx_n)} \varphi(t)dt = 0$ whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$. Jungck-Rhoades [13] obtained the concept of weakly compatible as follows:

Definition 2.9. Let S and T be self maps of a set X. If $w = fx = gx$ for some x in X, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition 2.10. Let *A* and *S* be mappings on *d*-metric space (*X*, *d*), then *A* and *S* are said to be weakly compatible mappings if they commute at their coincident points such that *Ax* = *Sx* implies *ASx* = *SAx*. The point *x* ∈ *X* is called coincident point of *A* and *S*. It is easy to see that compatible mapping commute at their coincidence points.

Lemma 2.1. Let A and B be weakly compatible self maps of a set X. If A and B have a unique point of coincidence $w = Ax = Bx$, then w is the unique common fixed point of A and B.

Example 2.2 .

Let *X* = [0, 3] be equipped with the usual *d*- metric space $d(x, y) = |x - y|$.

Define *S, T* : [0, 3] → [0, 3] by

$Sx = \begin{cases} x, & x \in [0,1) \\ 3, & x \in [1,3] \end{cases}$ and $Tx = \begin{cases} 3-x, & x \in [0,1) \\ 3, & x \in [1,3] \end{cases}$

Then for any $x \in [1,3]$, $STx = TSx$, showing that *S* and *T* are weakly compatible maps on [0, 3].

Proposition 2.1 Let *S* and *T* be compatible mappings from a *d*-metric space (X,d) into itself. Suppose that

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some $x \in X$.

if *S* is continuous then $\lim_{n\to\infty} TSx_n = Sx$.

Theorem 2.2 Let (X,d) be a complete d-metric space and let $T: X \to X$ be a contraction mapping, then *T* has a unique fixed point.

Definition 2.11. The function $\emptyset : R^+ \to R^+$ is called subadditive integrable function if and only if for all $a, b \in R^+$,

$$\int_0^{a+b} \emptyset(t)dt \le \int_0^a \emptyset(t)dt + \int_0^b \emptyset(t)dt$$

Example 2.3. Let $d(x,y) = |x-y|$and $\varphi(t) = \frac{1}{1+t}$ for all $t > 0$. Then for all $a, b \in R^+$,

$\int_0^{a+b} \frac{dt}{t+1} = ln(a+b+1)$, $\int_0^a \frac{dt}{t+1} = ln(a+1)$, $\int_0^b \frac{dt}{t+1} = ln(b+1)$,

Since $ab \ge 0$, then $a+b+1 \le a+b+1+ab = (a+1)(b+1)$.

Therefore, $ln(a+b+1) \le ln(a+1)(b+1) = ln(a+1) + ln(b+1)$ This shows that φ is an example of sub additive integrable function.

Definition 2.12. Let *A* and *B* be two self-mappings of a dislocated metric space (X,d). Then, *A* and *B* are said to be **occasionally weakly compatible (owc)** if there is a point $x \in X$ which is coincidence point of *A* and *B* at which *A* and *B* commute.

Example 2.4. Let us consider$X$ = [2.20] with the dislocated metric space (X,d) defined by $d(x,y) = (x-y)^2$. Define a self-map *A* and *B* by

$A(2) = 2$ at $x = 2$ and $A(x) = 6$ for $x > 2$

$B(2) = 2$ at $x = 2$, $B(x) = 12$ for $2 < x \le 5$ and $B(x) = x - 3$ for $x > 5$.

Now, $A(9) = B(9) = 6$, besides $x = 2$, $x = 9$ is another coincidence point of *A* and *B*.

$AB(2) = BA(2)$ but $AB(9) = 6, BA(9) = 3$, $AB(9) \ne BA(9)$ Therefore *A* and *B* are owc but not weakly compatible. Hence weakly compatible mappings are owc but not conversely.

Lemma 2.2. Let (X,d) be a semi-metric space. If the self mappings *A* and *B* on X have a unique point of coincidence *w*=Ax=Bx, then *w* is the unique common fixed point of *A* and *B*.

3.Main Results

We establish a some common fixed point results of integral type contractive conditions using the concept of compatibility and commutativity in complete dislocated metric (*d-metric*) spaces, which improves and extends similar known results in the literature.

Theorem 3.1 Let (X,d) be a complete dislocated metric space. Let $A, B, S, T, P, Q: X \to X$ satisfying the following conditions

(i) $AB(X) \subseteq Q(X)$ and $ST(X) \subseteq P(X)$ …(3.1)

(ii) The pairs (AB, P) or (ST, Q) are compatible. -----(3.2)

(iii) $$\int_0^{d(ABx,STy)} \emptyset(t)dt \le \alpha \int_0^{d(Px,ABx)} \emptyset(t)dt + \beta \int_0^{d(Qy,STy)} \emptyset(t)dt + \gamma \int_0^{d(Px,Qy)} \emptyset(t)dt + \mu \int_0^{d(Px,STy)} \emptyset(t)dt + \delta \int_0^{d(ABx,Qy)} \emptyset(t)dt \quad \text{------(3.3)}$$

for all $(x,y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$. … (3.4)

where $\alpha, \beta, \gamma, \mu, \delta \ge 0$, $0 \le \alpha + \beta + \gamma + 2\mu + 2\delta < 1$

If any one of AB, ST, P and Q is continuous for all $x, y \in X, then$ AB, ST, P and Q have a unique common fixed point in *X*. Furthermore, if the pairs (A,B), (A,P), (B,P), (S,T), (S,Q) and (T,Q) are commuting mappings then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Using condition (3.1), we define sequences $\{\boldsymbol{x_n}\}$ and $\{\boldsymbol{y_n}\}$ in $\boldsymbol{X}$ by the rule

$y_{2n+1} = Qx_{2n+1} = ABx_{2n}$ and $y_{2n} = Px_{2n} = STx_{2n-1}$ …(3.5)

Assume also that $y_{2n} \ne y_{2n+1}$ for all n

Now

$$\int_0^{d(y_{2n+1},y_{2n})} \emptyset(t)dt = \int_0^{d(ABx_{2n},STx_{2n-1})} \emptyset(t)dt$$

$$\le \alpha \int_0^{d(Px_{2n},ABx_{2n})} \emptyset(t)dt + \beta \int_0^{(Qx_{2n-1},STx_{2n-1})} \emptyset(t)dt + \gamma \int_0^{d(Px_{2n},Qx_{2n-1})} \emptyset(t)dt + \mu \int_0^{d(Qx_{2n},STx_{2n-1})} \emptyset(t)dt + \delta \int_0^{d(ABx_{2n},Qx_{2n-1})} \emptyset(t)dt$$

$$\le \alpha \int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt + \beta \int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt + \gamma \int_0^{d(y_{2n},y_{2n-1})} \emptyset(t)dt + \mu \int_0^{d(y_{2n},y_{2n})} \emptyset(t)dt + \delta \int_0^{d(y_{2n+1},y_{2n-1})} \emptyset(t)dt$$

Hence

$\int_0^{d(y_{2n+1},y_{2n})} \emptyset(t)dt \leq \frac{(\beta+\gamma+2\mu+\delta)}{1-\alpha-\delta} \int_0^{d(y_{2n},y_{2n-1})} \emptyset(t)dt$

$\Rightarrow \int_0^{d(y_{2n+1},y_{2n})} \emptyset(t)dt \leq h \int_0^{d(y_{2n},y_{2n-1})} \emptyset(t)dt$

where

$$h = \frac{(\beta+\gamma+2\mu+\delta)}{1-\alpha-\delta} < 1$$

This shows that

$\int_0^{d(y_{n+1},y_n)} \emptyset(t)dt \leq h \int_0^{d(y_n,y_{n-1})} \emptyset(t)dt \leq h^2 \int_0^{d(y_{n-1},y_{n-2})} \emptyset(t)dt \leq \cdots \leq h^n \int_0^{d(y_1,y_0)} \emptyset(t)dt$

For every integer $q > 0$ we have

$\int_0^{d(y_{n+q},y_n)} \emptyset(t)dt \leq \int_0^{d(y_{n+q},y_{n+q-1})} \emptyset(t)dt + \cdots + \int_0^{d(y_{n+2},y_{n+1})} \emptyset(t)dt + \int_0^{d(y_{n+1},y_n)} \emptyset(t)dt$

$\leq (h^{q-1} + \cdots + h^2 + h + 1) \int_0^{d(y_{n+1},y_n)} \emptyset(t)dt$

$\leq (h^{q-1} + \cdots + h^2 + h + 1) h^n \int_0^{d(y_1,y_0)} \emptyset(t)dt$

Hence $h < 1$, so $h^n \to 0$ as $n \to \infty$.

Therefore, $\int_0^{d(y_{n+q},y_n)} \emptyset(t)dt \to 0 \Rightarrow d(y_{n+q}, y_n) \to 0$, as $n \to \infty$.

This implies that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, so there exists a point $z \in X$ such that $\{y_n\} \to z$.

Consequently subsequences

$\{ABx_{2n}\}, \{Px_{2n}\}, \{STx_{2n-1}\}$ and $\{Qx_{2n+1}\} \to z \in X$ …(3.6)

Let us assume that *P* is continuous. Since the pair (AB, P) is compatible on *X* so by proposition 2.1 we have

P^2x_{2n} and $ABPx_{2n} \to Pz$ as $n \to \infty$. …(3.7)

Now consider

$\int_0^{d(ABx_{2n},STx_{2n-1})} \emptyset(t)dt \leq \alpha \int_0^{d(P^2x_{2n},ABPx_{2n})} \emptyset(t)dt + \beta \int_0^{d(Qx_{2n-1},STx_{2n-1})} \emptyset(t)dt + \gamma \int_0^{d(P^2x_{2n},Qx_{2n-1})} \emptyset(t)dt +$

$\mu \int_0^{d(P^2x_{2n},STx_{2n-1})} \emptyset(t)dt + \delta \int_0^{d(ABPx_{2n},Qx_{2n-1})} \emptyset(t)dt$

Now taking limit as $n \to \infty$ and using conditions (3.6) and (3.7) we have

$d(Pz, z) \leq \gamma d(Pz, z) + \mu d(Pz, z) + \delta d(Pz, z) = (\gamma + \mu + \delta) d(Pz, z)$,

which is a contradiction, since $(\gamma + \mu + \delta) \neq 1$.

Hence $d(Pz, z) = 0 \Rightarrow Pz = z$.

Now we show that z is fixed point of AB. For this consider

$\int_0^{d(ABz,STx_{2n-1})} \emptyset(t)dt \leq \alpha \int_0^{d(Pz,ABz)} \emptyset(t)dt + \beta \int_0^{d(Qx_{2n-1},STx_{2n-1})} \emptyset(t)dt + \gamma \int_0^{d(Pz,Qx_{2n-1})} \emptyset(t)dt +$

$\mu \int_0^{d(Pz,STx_{2n-1})} \emptyset(t)dt + \delta \int_0^{d(ABz,Qx_{2n-1})} \emptyset(t)dt$

$= \alpha \int_0^{d(z,ABz)} \emptyset(t)dt + \beta \int_0^{d(Qx_{2n-1},STx_{2n-1})} \emptyset(t)dt + \int_0^{d(z,Qx_{2n-1})} \emptyset(t)dt +$

$\mu \int_0^{d(z,STx_{2n-1})} \emptyset(t)dt + \delta \int_0^{d(ABz,Qx_{2n-1})} \emptyset(t)dt$

Taking limit as $n \to \infty$ we have,

$\int_0^{d(ABz,z)} \emptyset(t)dt \leq \alpha \int_0^{d(z,ABz)} \emptyset(t)dt + \delta \int_0^{d(ABz,z)} \emptyset(t)dt = (\alpha + \delta) \int_0^{d(ABz,z)} \emptyset(t)dt$,

which is a contradiction, since $(\alpha + \delta) \neq 1$. Therefore $d(ABz, z) = 0 \Rightarrow ABz = z$.

As $AB(X) \subset Q(X)$, so there exists a point $u \in X$ such that $z = ABz = Qu$.

Consider

$\int_0^{d(z,STu)} \emptyset(t)dt = \int_0^{d(ABz,STu)} \emptyset(t)dt$

$\leq \alpha \int_0^{d(Qz,ABz)} \emptyset(t)dt + \beta \int_0^{d(Qu,STu)} \emptyset(t)dt + \gamma \int_0^{d(Pz,Qu)} \emptyset(t)dt + \mu \int_0^{d(Pz,STu)} \emptyset(t)dt + \delta \int_0^{d(ABz,Qu)} \emptyset(t)dt$

$= \alpha \int_0^{d(z,z)} \emptyset(t)dt + \beta \int_0^{d(z,STu)} \emptyset(t)dt + \gamma \int_0^{d(z,z)} \emptyset(t)dt + \mu \int_0^{d(z,STu)} \emptyset(t)dt + \delta \int_0^{d(z,z)} \emptyset(t)dt$

$\leq (2\alpha + \beta + 2\gamma + \mu + 2\delta) \int_0^{d(z,STu)} \emptyset(t)dt$,

which is a contradiction since $(2\alpha + \beta + 2\gamma + \mu + 2\delta) \neq 1$.

Hence $d(z, STu) = 0 \Rightarrow z = STu$. By above relations, we obtain

$z = ABz = Qu = Pu = STu$

Since the pair (ST, Q) is compatible on X, so $d(STQu, QSTu) = 0 \Rightarrow STQu = QSTu$. Hence, $STz = Qz$.

Now we show that z is the fixed point of *Q*.

For this consider

$\int_0^{d(z,Qz)} \emptyset(t)dt = \int_0^{d(ABz,STz)} \emptyset(t)dt$

$\leq \alpha \int_0^{d(Qz,ABz)} \emptyset(t)dt + \beta \int_0^{d(Qz,STz)} \emptyset(t)dt + \gamma \int_0^{d(Pz,Qz)} \emptyset(t)dt + \mu \int_0^{d(Pz,STz)} \emptyset(t)dt + \delta \int_0^{d(ABz,Qz)} \emptyset(t)dt$

$= \alpha \int_0^{d(z,z)} \emptyset(t)dt + \beta \int_0^{d(Qz,Qz)} \emptyset(t)dt + \gamma \int_0^{d(z,Qz)} \emptyset(t)dt + \mu \int_0^{d(z,Qz)} \emptyset(t)dt + \delta \int_0^{d(z,Qz)} \emptyset(t)dt$

$= 2\alpha \int_0^{d(z,Qz)} \emptyset(t)dt + 2\beta \int_0^{d(z,Qz)} \emptyset(t)dt + \gamma \int_0^{d(z,Qz)} \emptyset(t)dt + \mu \int_0^{d(z,Qz)} \emptyset(t)dt + \delta \int_0^{d(z,Qz)} \emptyset(t)dt$

$\leq (2\alpha + 2\beta + \gamma + \mu + \delta) \int_0^{d(z,STu)} \emptyset(t)dt$

which is a contradiction, since $(2\alpha + 2\beta + \gamma + \mu + \delta) \neq 1$.

Hence. $d(z, Qz) = 0 \Rightarrow Qz = z$. Therefore $Pz = Qz = STz = ABz = z$. Hence, z is the common fixed point of the mappings $AB, ST, P\ and\ Q.$

Uniqueness: Let z and v be two common fixed point of the mappings $AB, ST, P\ and\ Q.$ Now by condition 3 we have

$\int_0^{d(z,v)} \emptyset(t)dt \ = \int_0^{d(ABz,STv)} \emptyset(t)dt$

$\leq\ \alpha \int_0^{d(Pz,ABz)} \emptyset(t)dt + \beta \int_0^{d(Qv,STv)} \emptyset(t)dt + \gamma \int_0^{d(Pz,Qv)} \emptyset(t)dt + \mu\ \int_0^{d(Pz,STv)} \emptyset(t)dt + \delta \int_0^{d(ABz,Qv)} \emptyset(t)dt$

$=\ \alpha \int_0^{d(z,z)} \emptyset(t)dt + \beta \int_0^{d(v,v)} \emptyset(t)dt + \gamma \int_0^{d(z,v)} \emptyset(t)dt +\ \mu\ \int_0^{d(z,v)} \emptyset(t)dt + \delta \int_0^{d(z,v)} \emptyset(t)dt$

$\leq (2\alpha + 2\beta + \gamma + \mu + \delta) \int_0^{d(z,v)} \emptyset(t)dt$

which is a contradiction since $(2\alpha + 2\beta + \gamma + \mu + \delta) = 1$.

So $d(z, v) = 0\ \Rightarrow z = v$. Thus z is the unique common fixed point of the mappings $AB, ST, P\ and\ Q.$

Finally, we prove that z is also a common fixed point of $A, B, S, T, P\ and\ Q$. Let both the pairs (AB, P) and (ST, Q) have a unique common fixed point z.

$Az = A(ABz) = A(BAz) = AB(Az)$, $Az = A(Pz) = P(Az)$

$Bz = B(ABz) = B\big(A(Bz)\big) = BA(Bz) = AB(Bz)$, $Bz = B(Pz) = P(Bz)$,

which implies that (AB, P) has common fixed points which are Az and Bz.

We get thereby $Az = z = Bz = Pz = ABz$.

Similarly, using the commutativity of $(S, T), (S, Q)$ and (T, Q), $Sz = z = Tz = Qz = STz$ can be shown.

Now, we need to show that $Az = Sz$ $(Bz = Tz)$.

By using condition (3.3), we have

$\int_0^{d(Az,Sz)} \emptyset(t)dt = \int_0^{d(A(BAz),S(TSz))} \emptyset(t)dt$

$= \int_0^{d(AB(Az),ST(Sz))} \emptyset(t)dt$

$\leq\ \alpha \int_0^{d(PAz,ABAz)} \emptyset(t)dt + \beta \int_0^{d(QSz,STSz)} \emptyset(t)dt + \gamma \int_0^{d(PAz,QSz)} \emptyset(t)dt +$

$\mu\ \int_0^{d(PSz,STSz)} \emptyset(t)dt + \delta \int_0^{d(ABAz,QSz)} \emptyset(t)dt$

$=\ \alpha \int_0^{d(Az,Az)} \emptyset(t)dt + \beta \int_0^{d(Sz,Sz)} \emptyset(t)dt + \gamma \int_0^{d(Az,Sz)} \emptyset(t)dt + \mu\ \int_0^{d(Az,Sz)} \emptyset(t)dt +$

$\delta \int_0^{d(Az,Sz)} \emptyset(t)dt$

$\leq (2\alpha + 2\beta + \gamma + \mu + \delta) \int_0^{d(Az,Sz)} \emptyset(t)dt,$

which is a contradiction since $(2\alpha + 2\beta + \gamma + \mu + \delta) \neq 1$.

Hence,

$\int_0^{d(Az,Sz)} \emptyset(t)dt = 0 \Rightarrow d(Az, Sz) = 0 \Rightarrow z = Sz.$

Similarly, $Bz = Tz$ can be shown.

Consequently, z is a unique common fixed point of $A, B, S, T, P\ and\ Q$.

This completes the proof of the theorem.

Now we have the following corollaries

If we put $AB = A$ and $ST = B$ in the above Theorem 3.1, then the theorem is reduced to the following corollary.

Corollary 3.1 Let (X, d) be a complete dislocated metric space. Let $A, B, P, Q: X \to X$ satisfying the following
conditions

(i) $A(X) \subseteq Q(X)$ and $B(X) \subseteq P(X)$

(ii) The pairs (A, P) or (B, Q) are compatible.

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt\ \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)dt +\ \beta \int_0^{d(Qy,By)} \emptyset(t)dt +\ \gamma \int_0^{d(Px,Qy)} \emptyset(t)dt + \mu \int_0^{d(Px,By)} \emptyset(t)dt +$
$\delta \int_0^{d(Ax,Qy)} \emptyset(t)dt$

for all $(x, y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

where $\alpha, \beta, \gamma, \mu, \delta \geq 0$, $0 \leq \alpha + \beta + \gamma + 2\mu + 2\delta < 1$

If any one of A, B, P and Q is continuous for all $x, y \in X$, $then$ A, B, P and Q have a unique common fixed point in X.

If we put Q = P in the above Corollary 3.1, then the theorem is reduced to the following corollary

Corollary 3.2 Let (X, d) be a complete dislocated metric space. Let $A, B, P: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq P(X)$ and $B(X) \subseteq P(X)$

(ii) The pairs (A, P) or (B, P) are compatible

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)dt + \beta \int_0^{d(Py,By)} \emptyset(t)dt + \gamma \int_0^{d(Px,Py)} \emptyset(t)dt + \mu \int_0^{d(Px,By)} \emptyset(t)dt + \delta \int_0^{d(Ax,Py)} \emptyset(t)dt$

for all $(x,y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

where $\alpha, \beta, \gamma, \mu, \delta \geq 0$, $0 \leq \alpha + \beta + \gamma + 2\mu + 2\delta < 1$.

If any one of A, B and P is continuous for all $x, y \in X$, then A, B and P have a unique common fixed point in *X*.

If we put $B = A$ in Corollary 3.1, then we obtain the following corollary.

Corollary 3.3 Let (X,d) be a complete dislocated metric space. Let $A, B, I: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq Q(X)$ and $A(X) \subseteq P(X)$

(ii) The pairs (A,P) or (A,Q) are compatible.

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)dt + \beta \int_0^{d(Qy,Ay)} \emptyset(t)dt + \int_0^{d(Px,Qy)} \emptyset(t)dt + \int_0^{d(Px,Ay)} \emptyset(t)dt + \delta \int_0^{d(Ax,Qy)} \emptyset(t)dt$

for all $(x,y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

where $\alpha, \beta, \gamma, \mu, \delta \geq 0$, $0 \leq \alpha + \beta + \gamma + 2\mu + 2\delta < 1$.

If any one of A,P and Q is continuous for all $x, y \in X, then$ A,P and Q have a unique common fixed point in *X*.

If we put $Q = P$ and $B = A$ in the above Corollary 3.1,then we obtain the following corollary

Corollary 3.4 Let (X,d) be a complete dislocated metric space. Let $A, P: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq P(X)$

(ii) The pairs (A,P) are compatible.

(iii) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \leq \alpha \int_0^{d(Px,Ax)} \emptyset(t)dt + \beta \int_0^{d(Py,Ay)} \emptyset(t)dt + \gamma \int_0^{d(Px,Py)} \emptyset(t)dt + \int_0^{d(Px,Ay)} \emptyset(t)dt + \delta \int_0^{d(Ax,Py)} \emptyset(t)dt$

for all $(x,y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

where $\alpha, \beta, \gamma, \mu, \delta \geq 0$, $0 \leq \alpha + \beta + \gamma + 2\mu + 2\delta < 1$

If any one of A and P is continuous for all $x, y \in X, then$ A and P have a unique common fixed point in *X*.

If we put $Q = P = I$ in the above Corollary 3.1, then the theorem is reduced to the following corollary.

Corollary 3.5 Let (X,d) be a complete dislocated metric space. Let $A, B, I: X \to X$ satisfying the following conditions

(i) $A(X) \subseteq X$ and $B(X) \subseteq X$

(ii) The pairs (A,I) or (B,I) are compatible.

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq \alpha \int_0^{d(x,Ax)} \emptyset(t)dt + \beta \int_0^{d(y,By)} \emptyset(t)dt + \int_0^{d(x,y)} \emptyset(t)dt + \mu \int_0^{d(x,By)} \emptyset(t)dt + \delta \int_0^{d(x,y)} \emptyset(t)dt$

for all $(x,y) \in X \times X$ where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

where $\alpha, \beta, \gamma, \mu, \delta \geq 0$, $0 \leq \alpha + \beta + \gamma + 2\mu + 2\delta < 1$.

If any one of A and B is continuous for all $x, y \in X$, then A and B have a unique common fixed point in *X*.

Now we establish a some common fixed point results of Ciric's [4] type contractive conditions using the concept of weak compatibility and commutativity in complete dislocated metric (*d-metric*) spaces, which improves and extends similar known results in the literature.

Theorem 3.2 Let (X,d) be a complete dislocated metric space. Let $A, B, S, T, P, Q: X \to X$ satisfying the following conditions

(i) $S(X) \subseteq PQ(X)$ and $T(X) \subseteq AB(X)$ …(3.8)

(ii) The pairs (S, AB) or (T, PQ) are weakly compatible. -----(3.9)

(iii) $\int_0^{d(Sx,Ty)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(ABx,Ty)} \emptyset(t)dt, \int_0^{d(PQy,Ty)} \emptyset(t)dt, \int_0^{d(ABx,Sx)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABx,PQy)+d(PQy,Sx)]} \emptyset(t)dt\right\}$ -----(3.10)

for all $(x,y) \in X \times X$ and $0 \leq h < \frac{1}{2}$,where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\in > 0$. … (3.11)

If any one of AB, ST, P and Q is continuous for all $x, y \in X$, then, AB, ST, P and Q have a unique common fixed point in *X*. Furthermore, if the pairs (A,B), (B,S), (P,Q) and (T,Q) are commuting mappings then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Using condition (3.8) we define sequences $\{\boldsymbol{x_n}\}$ and $\{\boldsymbol{y_n}\}$ in $\boldsymbol{X}$ by the rule

$y_{2n+1} = ABx_{2n+2} = Tx_{2n+2}$ and $y_{2n} = PQx_{2n+1} = Sx_{2n}$ …(3.12)

Assume also that $y_{2n} \neq y_{2n+1}$ for all n

Now

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt = \int_0^{d(Sx_{2n},Tx_{2n+1})} \emptyset(t)dt$$

$$\leq h.max\left\{\left\{\int_0^{d(ABx_{2n},Tx_{2n+1})} \emptyset(t)dt, \int_0^{d(PQx_{2n+1},Tx_{2n+1})} \emptyset(t)dt, \int_0^{d(ABx_{2n},Sx_{2n})} \emptyset(t)dt,\right.\right.$$

$$\left.\int_0^{\frac{1}{2}[d(ABx_{2n},PQx_{2n+1})+d(PQx_{2n+1},Sx_{2n})]} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\left\{\int_0^{d(y_{2n-1},y_{2n+1})} \emptyset(t)dt, \int_0^{d(y_{2n},y_{2n+1})))} \emptyset(t)dt, \int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt,\right.\right.$$

$$\left.\int_0^{\frac{1}{2}[d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n})]} \emptyset(t)dt,\right\}$$

$$\leq h.max\left\{\int_0^{d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n+1})} \emptyset(t)dt, \int_0^{d(y_{2n},y_{2n+1}))} \emptyset(t)dt, \int_0^{d(y_{2n-1},y_{2n}))} \emptyset(t)dt,\right.$$

$$\left.\int_0^{\frac{1}{2}[d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n+1})+d(y_{2n+1},y_{2n})]} \emptyset(t)dt\right\}$$

Or

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq h\left[\int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt + \int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt\right],$$

which implies that

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq \frac{h}{1-h}\int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt$$

Let $k = \frac{h}{1-h} < 1$,

So

$$\int_0^{d(y_{2n},y_{2n+1})} \emptyset(t)dt \leq k\int_0^{d(y_{2n-1},y_{2n})} \emptyset(t)dt$$

$$\Rightarrow \int_0^{d(y_{2n+1},y_{2n})} \emptyset(t)dt \leq h\int_0^{d(y_{2n},y_{2n-1})} \emptyset(t)dt$$

This shows that

$$\int_0^{d(y_{n+1},y_n)} \emptyset(t)dt \leq h\int_0^{d(y_n,y_{n-1})} \emptyset(t)dt \leq h^2\int_0^{d(y_{n-1},y_{n-2})} \emptyset(t)dt \leq \cdots \leq h^n\int_0^{d(y_1,y_0)} \emptyset(t)dt$$

For every integer $q > 0$ we have

$$\int_0^{d(y_{n+q},y_n)} \emptyset(t)dt \leq \int_0^{d(y_{n+q},y_{n+q-1})} \emptyset(t)dt + \cdots + \int_0^{d(y_{n+2},y_{n+1})} \emptyset(t)dt + \int_0^{d(y_{n+1},y_n)} \emptyset(t)dt$$

$$\leq (h^{q-1} + \cdots + h^2 + h + 1)\int_0^{d(y_{n+1},y_n)} \emptyset(t)dt$$

$$\leq (h^{q-1} + \cdots + h^2 + h + 1)h^n\int_0^{d(y_1,y_0)} \emptyset(t)dt$$

Hence $h < 1$, so $h^n \to 0$ as $n \to \infty$.

Therefore, $\int_0^{d(y_{n+q},y_n)} \emptyset(t)dt \to 0 \Rightarrow d(y_{n+q}, y_n) \to 0$, as $n \to \infty$.

This implies that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, so there exists a point $z \in X$ such that $\{y_n\} \to z$.

Also the sub-sequences $\{y_{2n}\}$ and $\{y_{2n+1}\}$ converges to z as n tends to infinity. Therefore

$\{Sx_{2n}\}, \{PQx_{2n+1}\}, \{Tx_{2n+1}\}$ and $\{ABx_{2n+2}\} \to z \in X$ …(3.13)

Since $T(X) \subseteq AB(X)$, there exists a point $u \in X$ such that $ABu = z$. We show that

$$ABu = Su = z$$

From condition (3.10), we have

$$\int_0^{d(Su,z)} \emptyset(t)dt = \int_0^{d(Su,Tx_{2n+1})} \emptyset(t)dt$$

$$\leq h.max\left\{\int_0^{d(ABu,Tx_{2n+1})} \emptyset(t)dt, \int_0^{d(PQx_{2n+1},Tx_{2n+1})} \emptyset(t)dt, \int_0^{d(ABu,Su)} \emptyset(t)dt,\right.$$

$$\left.\int_0^{\frac{1}{2}[d(ABu,PQx_{2n+1})+d(PQx_{2n+1},Su)]} \emptyset(t)dt\right\}$$

Taking limit as $n \to \infty$ we have

$$\int_0^{d(Su,z)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(z,z)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{d(z,Su)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,z)+d(z,Su)]} \emptyset(t)dt\right\}$$

$\leq h.max\Big\{\int_0^{d(z,Su)+d(z,Su)} \emptyset(t)dt , \int_0^{d(z,Su)+d(z,Su)} \emptyset(t)dt , \int_0^{d(z,Su)} \emptyset(t)dt,$
$\int_0^{\frac{1}{2}[d(z,Su)+d(z,Su)+d(z,Su)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{2\int_0^{d(z,Su)} \emptyset(t)dt , 2\int_0^{d(z,Su)} \emptyset(t)dt , \int_0^{d(z,Su)} \emptyset(t)dt , \frac{3}{2}\int_0^{d(z,Su)} \emptyset(t)dt\Big\}$
$\leq 2h\int_0^{d(z,Su)} \emptyset(t)dt$

Since $2h < 1$, so the above inequality is possible only if,
$d(Su, z) = 0 \Rightarrow Su = z$
Thus $ABu = Su = z$
Again, since $S(X) \subseteq PQ(X)$, there exists a point $v \in X$ such that $PQv = z$. We show that $PQv = Tv = z$
From condition (3.10), we have
$\int_0^{d(z,Tv)} \emptyset(t)dt = \int_0^{d(Sx_{2n},Tv)} \emptyset(t)dt$
$\leq h.max\Big\{\int_0^{d(ABx_{2n},Tv)} \emptyset(t)dt , \int_0^{d(PQv,Tv)} \emptyset(t)dt ,$
$\int_0^{d(ABx_{2n},Sx_{2n})} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABx_{2n},PQv)+d(PQv,Sx_{2n})]} \emptyset(t)dt\Big\}$
Taking limit as $n \to \infty$ we have
$\int_0^{d(z,Tv)} \emptyset(t)dt \leq h.max\Big\{\int_0^{d(z,Tv)} \emptyset(t)dt , \int_0^{d(z,Tv)} \emptyset(t)dt , \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,z)+d(z,z)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(z,Tv)} \emptyset(t)dt , \int_0^{d(z,Tv)} \emptyset(t)dt , \int_0^{d(z,Tv)+d(z,Tv)} \emptyset(t)dt, \int_0^{[d(z,Tv)+d(z,Tv)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(z,Tv)} \emptyset(t)dt , \int_0^{d(z,Tv)} \emptyset(t)dt , 2\int_0^{d(z,Tv)} \emptyset(t)dt , 2\int_0^{d(z,Tv)} \emptyset(t)dt\Big\}$
$\leq 2h\int_0^{d(z,Tv)} \emptyset(t)dt$
Since $2h < 1$, so the above inequality is possible only if,
$d(z, Tv) = 0 \Rightarrow Tv = z$
Thus $PQv = Tv = z$
Hence, $ABu = Su = Tv = PQv = z$.
Since, the pair (S, AB) is weakly compatible, so $SABu = ABSu \Rightarrow Sz = ABz$.
Now, we show that z is a fixed point of S in the following:
$\int_0^{d(Sz,z)} \emptyset(t)dt = \int_0^{d(Sz,Tv)} \emptyset(t)dt$
$\leq h.max\Big\{\int_0^{d(ABz,Tv)} \emptyset(t)dt , \int_0^{d(PQv,Tv)} \emptyset(t)dt ,$
$\int_0^{d(ABz,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABz,PQv)+d(PQv,Sz)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(Sz,z)} \emptyset(t)dt , \int_0^{d(z,z)} \emptyset(t)dt , \int_0^{d(Sz,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Sz,z)+d(Sz,Sz)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(Sz,z)} \emptyset(t)dt , \int_0^{d(z,Sz)+d(Sz,z)} \emptyset(t)dt ,$
$\int_0^{d(Sz,z)+d(z,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Sz,z)+d(Sz,z)+d(z,Sz)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(Sz,z)} \emptyset(t)dt , 2\int_0^{d(Sz,z)} \emptyset(t)dt , 2\int_0^{d(Sz,z)} \emptyset(t)dt , \frac{3}{2}\int_0^{d(Sz,z)} \emptyset(t)dt\Big\}$
$\leq 2h\int_0^{d(Sz,z)} \emptyset(t)dt$
Since $2h < 1$, so the above inequality is possible only if,
$d(Sz, z) = 0 \Rightarrow Sz = z$
Thus $ABz = Sz = z$
Again, the pair (T, PQ) is weakly compatible, so $TPQv = PQTv \Rightarrow Tz = PQz$.
Now, we show that z is a fixed point of T as:
$\int_0^{d(z,Tz)} \emptyset(t)dt = \int_0^{d(Sz,Tz)} \emptyset(t)dt$
$\leq h.max\Big\{\int_0^{d(ABz,Tz)} \emptyset(t)dt , \int_0^{d(PQz,Tz)} \emptyset(t)dt ,$
$\int_0^{d(ABz,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABz,PQz)+d(PQz,Sz)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(z,Tz)} \emptyset(t)dt , \int_0^{d(Tz,Tz)} \emptyset(t)dt , \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,Tz)+d(Tz,z)]} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(z,Tz)} \emptyset(t)dt , \int_0^{d(Tz,z)+d(z,Tz)} \emptyset(t)dt , \int_0^{d(z,Tz)+d(Tz,z)} \emptyset(t)dt, \int_0^{d(z,Tz)} \emptyset(t)dt\Big\}$
$\leq h.max\Big\{\int_0^{d(z,Tz)} \emptyset(t)dt , 2\int_0^{d(z,Tz)} \emptyset(t)dt , 2\int_0^{d(z,Tz)} \emptyset(t)dt , \int_0^{d(z,Tz)} \emptyset(t)dt\Big\}$
$\leq 2h\int_0^{d(Sz,z)} \emptyset(t)dt$
Since $2h < 1$, so the above inequality is possible only if,

$d(z, Tz) = 0 \Rightarrow Tz = z$

Thus $ABz = PQz = Sz = Tz = z$

Hence, z is the common fixed point of the mappings *AB, PQ, S* and *T.*

Uniqueness: Let z and w be two common fixed point of the mappings *AB, ST, P* and *Q.* Now by condition 2 we have

$$\int_0^{d(z,w)} \emptyset(t)dt = \int_0^{d(Sz,Tw)} \emptyset(t)dt$$
$$\leq h.max\Big\{\int_0^{d(ABz,Tw)} \emptyset(t)dt, \int_0^{d(PQw,Tw)} \emptyset(t)dt, \int_0^{d(ABz,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABz,PQw)+d(PQw,Sz)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,w)} \emptyset(t)dt, \int_0^{d(w,w)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,w)+d(w,z)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,w)} \emptyset(t)dt, 2\int_0^{d(z,w)} \emptyset(t)dt, 2\int_0^{d(z,w)} \emptyset(t)dt, \int_0^{d(z,w)} \emptyset(t)dt\Big\}$$
$$\leq 2h\int_0^{d(z,w)} \emptyset(t)dt,$$

which is a contradiction, since $2h < 1$. So $\mathrm{d(z,w)} = 0$ implies $\mathrm{z = w}$. Thus z is the unique common fixed point of the mappings AB, PQ, S and T.

Now, we need to show that $Bz = Qz$.

By using condition (2), we have

$$\int_0^{d(Bz,Qz)} \emptyset(t)dt = \int_0^{d(BSz,QTz)} \emptyset(t)dt = \int_0^{d(SBz,TQz)} \emptyset(t)dt$$
$$\leq h.max\Big\{\int_0^{d(ABBz,TQz)} \emptyset(t)dt, \int_0^{d(PQQz,TQz)} \emptyset(t)dt, \int_0^{d(ABBz,SBz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABBz,PQQz)+d(PQQz,SBz)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(BABz,QTz)} \emptyset(t)dt, \int_0^{d(QPQz,QTz)} \emptyset(t)dt, \int_0^{d(BABz,BSz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(BABz,QPQz)+d(QPQz,BSz)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(Bz,Qz)} \emptyset(t)dt, \int_0^{d(Qz,Qz)} \emptyset(t)dt, \int_0^{d(Bz,Bz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Bz,Qz)+d(Qz,Bz)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(Bz,Qz)} \emptyset(t)dt, \int_0^{d(Qz,Bz)+d(Bz,Qz)} \emptyset(t)dt, \int_0^{d(Bz,Qz)+d(Qz,Bz)} \emptyset(t)dt, \int_0^{d(Bz,Qz)} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(Bz,Qz)} \emptyset(t)dt, 2\int_0^{d(Bz,Qz)} \emptyset(t)dt, 2\int_0^{d(Bz,Qz)} \emptyset(t)dt, \int_0^{d(Bz,Qz)} \emptyset(t)dt\Big\}$$
$$\leq 2h\int_0^{d(Bz,Qz)} \emptyset(t)dt,$$

which is a contradiction, since $2h < 1$.

Hence,

$d(Bz, Qz) = 0 \Rightarrow Bz = Qz$.

Now, we need to show that $z = Qz$.

By using condition (3.10), we have

$$\int_0^{d(z,Qz)} \emptyset(t)dt = \int_0^{d(Sz,QTz)} \emptyset(t)dt = \int_0^{d(Sz,TQz)} \emptyset(t)dt$$
$$\leq h.max\Big\{\int_0^{d(ABz,TQz)} \emptyset(t)dt, \int_0^{d(PQQz,TQz)} \emptyset(t)dt, \int_0^{d(ABz,Sz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABz,PQQz)+d(PQQz,Sz)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,QTz)} \emptyset(t)dt, \int_0^{d(QPQz,QTz)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,QPQz)+d(QPQz,z)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,Qz)} \emptyset(t)dt, \int_0^{d(Qz,Qz)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(z,Qz)+d(Qz,z)]} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,Qz)} \emptyset(t)dt, \int_0^{d(Qz,z)+d(z,Qz)} \emptyset(t)dt, \int_0^{d(z,Qz)+d(Qz,z)} \emptyset(t)dt, \int_0^{d(z,Qz)} \emptyset(t)dt\Big\}$$
$$\leq h.max\Big\{\int_0^{d(z,Qz)} \emptyset(t)dt, 2\int_0^{d(z,Qz)} \emptyset(t)dt, 2\int_0^{d(z,Qz)} \emptyset(t)dt, \int_0^{d(z,Qz)} \emptyset(t)dt\Big\}$$
$$\leq 2h\int_0^{d(z,Qz)} \emptyset(t)dt,$$

which is a contradiction, since $2h < 1$.
Hence,
$d(z, Qz) = 0 \Rightarrow z = Qz$.
Now, we need to show that $Bz = z$.
By using condition (3.10), we have

$$\int_0^{d(Bz,z)} \emptyset(t)dt = \int_0^{d(BSz,Tz)} \emptyset(t)dt = \int_0^{d(SBz,Tz)} \emptyset(t)dt$$

$$\leq h.max\left\{\int_0^{d(ABBz,Tz)} \emptyset(t)dt, \int_0^{d(PQz,Tz)} \emptyset(t)dt, \int_0^{d(ABBz,SBz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(ABBz,PQz)+d(PQz,SBz)]} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(BABz,z)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{d(BABz,BSz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(BABz,z)+d(z,BSz)]} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(Bz,z)} \emptyset(t)dt, \int_0^{d(z,z)} \emptyset(t)dt, \int_0^{d(Bz,Bz)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Bz,z)+d(z,Bz)]} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(Bz,z)} \emptyset(t)dt, \int_0^{d(z,Bz)+d(Bz,z)} \emptyset(t)dt, \int_0^{d(Bz,z)+d(z,Bz)} \emptyset(t)dt, \int_0^{d(Bz,z)} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(Bz,z)} \emptyset(t)dt, 2\int_0^{d(Bz,z)} \emptyset(t)dt, 2\int_0^{d(Bz,z)} \emptyset(t)dt, \int_0^{d(Bz,z)} \emptyset(t)dt\right\}$$

$$\leq 2h\int_0^{d(Bz,z)} \emptyset(t)dt,$$

which is a contradiction, since $2h < 1$. Hence $d(Bz, z) = 0 \Rightarrow Bz = z$.
Thus $PQz = Pz = Qz = z$ and $ABz = Az = Bz = z$
Therefore $Az = Bz = Sz = Tz = Pz = Qz = z$
Consequently, z is a unique common fixed point of A, B, S, T, P and Q.
This completes the proof of the theorem.
Now we have the following corollaries
If we put AB = A and PQ= B in the above Theorem 3.2, then it is reduced to the following corollary.

Corollary 3.6 Let (X, d) be a complete dislocated metric space. Let $A, B, S, T: X \to X$ satisfying the following conditions

(i) $S(X) \subseteq B(X)$ and $T(X) \subseteq A(X)$
(ii) The pairs (S, A) or (T, B) are weakly compatible.
(iii) $\int_0^{d(Sx,Ty)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(Ax,Ty)} \emptyset(t)dt, \int_0^{d(By,Ty)} \emptyset(t)dt, \int_0^{d(Ax,Sx)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Ax,By)+d(By,Sx)]} \emptyset(t)dt\right\}$

for all $(x, y) \in X \times X$ and $0 \leq h < \frac{1}{2}$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.
If any one of A, B, S and T is continuous for all $x, y \in X$, then A, B, S and T have a unique common fixed point in X.
If we put A = B in the above Corollary 3.6, then the theorem is reduced to the following corollary.

Corollary 3.7 Let (X, d) be a complete dislocated metric space. Let $A, S, T: X \to X$ satisfying the following conditions

(i) $S(X) \subseteq A(X)$ and $T(X) \subseteq A(X)$
(ii) The pairs (S, A) or (T, A) are weakly compatible.
(iii) $\int_0^{d(Sx,Ty)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(Ax,Ty)} \emptyset(t)dt, \int_0^{d(Ay,Ty)} \emptyset(t)dt, \int_0^{d(Ax,Sx)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Ax,Ay)+d(Ay,Sx)]} \emptyset(t)dt\right\}$

for all $(x, y) \in X \times X$ and $0 \leq h < \frac{1}{2}$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.
If any one of A, S and T is continuous for all $x, y \in X$, then A,S and T have a unique common fixed point in X.
If we put S = T in the above Corollary 3.6, then the theorem is reduced to the following corollary

Corollary 3.8 Let (X, d) be a complete dislocated metric space. Let $A, B, S: X \to X$ satisfying the following conditions

(i) $S(X) \subseteq B(X)$ and $S(X) \subseteq A(X)$
(ii) The pairs (S, A) or (S, B) are weakly compatible.
(iii) $\int_0^{d(Sx,Sy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(Ax,Sy)} \emptyset(t)dt, \int_0^{d(By,Sy)} \emptyset(t)dt, \int_0^{d(Ax,Sx)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Ax,By)+d(By,Sx)]} \emptyset(t)dt\right\}$

for all $(x, y) \in X \times X$ and $0 \leq h < \frac{1}{2}$, where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

If any one of A,B and S is continuous for all $x, y \in X,$ then A,B and S have a unique common fixed point in *X*.

If we put B = A and S = T in the above Corollary 3.6, it is reduced to the following corollary

Corollary 3.9 Let (X, d) be a complete dislocated metric space. Let $A, S: X \to X$ satisfying the following conditions

(i) $S(X) \subseteq A(X)$ and $S(X) \subseteq A(X)$

(ii) The pairs (S, A) is weakly compatible.

(iii) $\int_0^{d(Sx,Sy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(AxSy)} \emptyset(t)dt, \int_0^{d(Ay,Sy)} \emptyset(t)dt, \int_0^{d(Ax,Sx)} \emptyset(t)dt, \int_0^{\frac{1}{2}[d(Ax,Ay)+d(Ay,Sx)]} \emptyset(t)dt\right\}$

for all $(x, y) \in X \times X$ and $0 \leq h < \frac{1}{2}$,where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$.

If any one of A and S is continuous for all $x, y \in X,$ then A and S have a unique common fixed point in *X*.

Now we establish a common fixed point theorem in dislocated metric space with occasionally weakly compatible which improves and extends similar known results in the literature.

Theorem 3.3: Let *(X, d)* be a complete dislocated metric space. Let A, B, S, T, P and Q be self mappings of X such that

(i) $\{P, AB\}$ and $\{Q, ST\}$ are occasionally weakly compatible (owc), … (3.14)

(ii) $\int_0^{d(Px,Qy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(ABx,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,ABx)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,ABx)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,STy)} \emptyset(t)dt,\right\}$ ……. (3.15)

for all $(x, y) \in X \times X$,where $0 \leq h < 1$.

Then *AB*, *ST*, *P* and Q have a unique common fixed point. Furthermore, if *AB*= BA and ST= TS,then *A*, *B*, *T*, *S*, *P* and Q have a unique common fixed point.

Proof: Since $\{P, AB\}$ and $\{Q, ST\}$ are occasionally weakly compatible (owc), then there exists $(x, y) \in X \times X$such that $Px = ABx = x$,where $PABx = ABPx$ and $STy = Qy = y$, where, $STQy = QSTy$.We claim that $Px = Qy$. Using condition (ii), we get

$$\int_0^{d(Px,Qy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(ABx,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,ABx)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,ABx)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,STy)} \emptyset(t)dt,\right\} \quad ……. (3.15)$$

$$= h.max\left\{\int_0^{d(Px,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Px)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Px)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Qy)} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(Px,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Qy)+d(Qy,Px)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Px)+d(Px,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Px)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Qy)} \emptyset(t)dt\right\}$$

$$= h.max\left\{\int_0^{d(Px,Qy)} \emptyset(t)dt, \int_0^{d(Px,Qy)} \emptyset(t)dt, \int_0^{d(Px,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Qy)} \emptyset(t)dt\right\}$$

$$= h\int_0^{d(Px,Qy)} \emptyset(t)dt,$$

which is contradiction, since. $0 \leq h < 1$.

So, $\int_0^{d(Px,Qy)} \emptyset(t)dt = 0 \Rightarrow d(Px, Qy) = 0 \Rightarrow Px = Qy$

Therefore, $ABx = Px = STy = Qy$. … (3.16)

Moreover, if there is another point of coincidence z such that $z = Pz = ABz$. We claim that $Pz = Qy$. Using condition (ii), we get

$$\int_0^{d(Pz,Qy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(ABz,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,ABz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,STy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,ABz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,STy)} \emptyset(t)dt,\right\}$$

$$= h.max\left\{\int_0^{d(Pz,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Pz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Pz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Qy)} \emptyset(t)dt\right\}$$

$$\leq h.max\left\{\int_0^{d(Pz,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Qy)+d(Qy,Pz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Pz)+d(Pz,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Pz)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Qy)} \emptyset(t)dt\right\}$$

$$= h.max\left\{\int_0^{d(Pz,Qy)} \emptyset(t)dt, \int_0^{d(Pz,Qy)} \emptyset(t)dt, \int_0^{d(Pz,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Qy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pz,Qy)} \emptyset(t)dt\right\}$$

$$= h\int_0^{d(Pz,Qy)} \emptyset(t)dt,$$

which is contradiction,since. $0 \leq \mathrm{h} < 1$.
So, $\int_0^{d(Pz,Qy)} \emptyset(t)dt = 0 \Rightarrow d(Pz,Qy) = 0 \Rightarrow Pz = Qy$
Therefore,
$Pz = ABz = STy = Qy$. … (3.17)
Similarily, if there is another point of coincidence v such that $v = STv = Qv$. It can be easily seen that $Pz = Qv$. Therefore
$Pz = ABz = STv = Qv$.

Also from (3.16) and (3.17), it follows that $ABz = ABx$. This implies that $z = x$.Hence, $w = ABx = Px$, for $w \in X$, is the unique point of coincidence of AB and P. By Lemma 2.1, w is the unique common fixed point of AB and P. Hence $w = ABw = Pw$. Similarly, there is a unique common fixed point $u \in X$ such that $u = STu = Qu$.. Suppose that $w \neq u$.Then using condition (ii), we get.

$$\int_0^{d(w,u)} \emptyset(t)dt = \int_0^{d(Pw,Qu)} \emptyset(t)dt$$
$$\leq h.max\left\{\int_0^{d(ABw,STu)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pw,ABw)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qu,STu)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qu,ABw)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Pw,STu)} \emptyset(t)dt,\right\}$$
$$= h.max\left\{\int_0^{d(w,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,w)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(u,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(u,w)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,u)} \emptyset(t)dt\right\}$$
$$\leq h.max\left\{\int_0^{d(w,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,u)+d(u,w)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(u,w)+d(w,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(u,w)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,u)} \emptyset(t)dt\right\}$$
$$= h.max\left\{\int_0^{d(w,u)} \emptyset(t)dt, \int_0^{d(w,u)} \emptyset(t)dt, \int_0^{d(w,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,u)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(w,u)} \emptyset(t)dt\right\}$$
$$= h\int_0^{d(w,u)} \emptyset(t)dt,$$

which is contradiction,since. $0 \leq \mathrm{h} < 1$.
So, $\int_0^{d(w,u)} \emptyset(t)dt = 0 \Rightarrow d(w,u) = 0 \Rightarrow w = u$

Hence, w is the unique common fixed point of *AB*, *TS*, *P* and Q. Finally, we need to show that w is only the common fixed point of mappings A, B, T, S, P and Q.
Let both the pairs (P,AB) and (Q,ST) have a unique common fixed point w.
AB= BA ,then for this, we can write
$Aw = A(ABw) = A(BAw) = AB(Aw)$, $Aw = A(Pw) = P(Aw)$
$Aw = B(ABw) = B(A(Bw)) = BA(Bw) = AB(Bw)$, $Bw = B(Pw) = P(Bw)$,
which implies that (P,AB) has common fixed points which are Az and Bz. We get thereby $Aw = w = Bw = Pw = ABw$.Similarly, using the commutativity of (S,T),$Sw = w = Tw = Qw = STw$ can be shown.
Hence *A*, *B*, *T*, *S*, *P* and Q have a unique common fixed point.

On the basis of above Theorem 3.3, we have the following corollary.

In the above Theorem 3.3, if we take $AB = A, ST = S$, then we have the following corollary.

Corollary 3.10: Let *(X, d)* be a complete dislocated metric space. Let A, S, P and Q be self mappings of X such that
(i) $\{P,A\}$ and $\{Q,S\}$ are occasionally weakly compatible (owc),
(ii) $\int_0^{d(Px,Qy)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(Ax,Sy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Sy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Qy,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Sy)} \emptyset(t)dt,\right\}$
for all $(x,y) \in X \times X$,where $0 \leq \mathrm{h} < 1$.
Then A, S, P and Q have a unique common fixed point.

In Corollary 3.10, if we take $P = Q$, then we have the following corollary.

Corollary 3.11: Let *(X, d)* be a complete dislocated metric space. Let A, P and S be self mappings of X such that
(i) $\{P,A\}$ and $\{P,S\}$ are occasionally weakly compatible (owc),
(ii) $\int_0^{d(Px,Py)} \emptyset(t)dt \leq h.max\left\{\int_0^{d(Ax,Sy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Py,Sy)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Py,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Sy)} \emptyset(t)dt,\right\}$
for all $(x,y) \in X \times X$,where $0 \leq \mathrm{h} < 1$.
Then A, P and S have a unique common fixed point.

In Corollary 3.10, if we take $P = Q$, $A = S$ then we have the following corollary.

Corollary3.12: Let *(X, d)* be a complete dislocated metric space. Let *A* and *P* be self mappings of X such that
(i) *P* and A are occasionally weakly compatible (owc),

(ii) $\int_0^{d(Px,Py)} \emptyset(t)dt \le h.max\left\{\int_0^{d(Ax,Ay)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Py,Ay)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Py,Ax)} \emptyset(t)dt, \frac{1}{2}\int_0^{d(Px,Ay)} \emptyset(t)dt,\right\}$

for all $(x, y) \in X \times X$,where $0 \le h < 1$.

Then *A* and *P* have a unique common fixed point.

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DOI: https://doi.org/10.15379/ijmst.v10i1.3711