Theoretical Modeling of a Beam with Variable Section and Finite Rigidity in A Fatigue Testing Machine to Verify the Bending Moment Produced in The Central Portion of The Smallest Diameter of The Beam Subjected to External Loads

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Abstracts: There is a bending fatigue machine where it is detected that when loads are applied, the theoretical model of the beam with which the machine was designed to obtain the bending moment does not coincide with the real measurements of the bending moment obtained experimentally and the specimen does not break, to determine the correct theoretical modeling applicable to this fatigue machine, the following static models are proposed, and the theoretical results are obtained through computer simulation. The bending moment values produced by the applied loads are verified with measurements through the Hooke and Navier law using strain gages, the analysis of the different modeling is carried out and those that are closest to the real model obtained experimentally are analyzed.

Keywords: Fatigue, Beam, Hooke, Moment, Bending, Steel Specimen, Rigidity, Forces.

1. INTRODUCTION

The original design of the testing machine was conceived in such a way that the first modeling is fulfilled, which will be analyzed in point 3.1, this is the R. RMoore model, when carrying out the experimentation, it was possible to verify that, when placing a weight of 166.88 n, a moment of 2842.6 n*mm was transmitted, according to the theory of conception of the equipment, this moment should be 16688.0 n *mm, a value that is very far from experimentation, this is the reason why this analysis begins. The calculations that are presented in this document apply to the equipment in which this experimentation is carried out, and the modeling has been adjusted to determine the best approach to the reality of this particular equipment.

2. MATERIALS & METHODS

There is a rotary bending fatigue machine made up of two bearings, 1 cardan type support, 1 3000 rpm motor, 1/2hp, 110v, mechanical system to apply load, beam with finite rigidity or specimen made of ASTM A36 steel (see figure 1), this machine was designed so that the bending moment being calculated according to the simply supported beam model with symmetrical load. initially the shaft was connected to the engine through a flexible flange, with the desire to improve the load transmission conditions, the flexible flange was changed to a cardan, the results were improved and the calculations are shown in point 3 of this document.

To carry out the experimental measurement of the unitary deformation, an omega brand meter, model dmd-21 with a channel, is used, which measures the unitary deformations through the use of strain gages at a certain central point of the test beam, this equipment yields dimensionless values in epsilon measurements (see figure 2).

Regarding the method to be used to calculate the bending moment m. it is based on Hooke's law $\sigma=\varepsilon*E$, where, ε (epsilon) is the unit strain obtained from the measurement with the unit strain gauge, E (elastic modulus) of the 200 GPa specimen obtained from “Riley/Sturges/Morris materials mechanics”, and also from the Navier formula

$\sigma=(M*r)/I$ which, solving for M, results, $M = \frac{\sigma * I}{r}$, to calculate the inertia, the following expression is used: $I = \frac{\pi * d^4}{64}$.

Programs as Solid Works and RISA 2d will be used for the mechanical design analysis.

Figure 1. Flexural fatigue machine with mounted test specimen.

Figure 2. Strain measuring equipment.

3. RESULTS

The first thing that was carried out were the experimental measurements in order to have the values with which it will be compared theoretically and check the validity of the modeling. The initial calculations were carried out to help us to obtain the bending moment using Hooke's law, the measurements made in the rotary bending fatigue machine and specimen were used. The initial data are shown in Table 1.

Table 1. Initial data

d(mm)	7,87	Diameter
r(mm)	3,935	Radio
I(mm^4)	188,31	Inertia
E*I ($N*m^2$)	37,66	Flexural Rigiditi
E (MPa)	200000	Elasticity Modulus

The strain gage is placed at the middle of the external surface of the steel specimen and connected to the Omega DMD-21 meter. The specimen with strain gage is placed on the mandrels of the rotary bending fatigue machine applying different loads in the mechanical force application system, once the system is assembled, the unitary deformations ε are measured and the bending moment M is calculated. The following is an example of calculation for a load of 17.011 Kg (166.88 N). Table 2, shows the results that were obtained for the different load values.

$$\sigma = E * \epsilon$$

$$\sigma = 200000 * 297x10^{-6}$$

$$\sigma = 59{,}4 \text{ MPa}$$

$$M = \frac{\sigma * I}{r}$$

$$M = \frac{59{,}4 * 188{,}31}{3{,}935}$$

$$M = 2842{,}6 \text{ N} * \text{mm}$$

Table 2. Calculation of the bending moment for different loads

W (Kg.)	Eps. 1	Eps. 2	Eps. 3	Eps. Prom.	σ(MPa)	M(N*mm)
1,546	24	24	24	24,00	4,8	229,70
4,639	70	70	70	70,00	14	669,97
10,825	198	198	192	196,00	39,2	1875,91
17,011	297	297	297	297,00	59,4	2842,57

The values shown in Table 2, which are measured experimentally, are taken as reference values and which we must approach with theoretical modeling. For software calculations, the load of 17,011 Kg will be used.

3.1. Modeling 1.

In this first modeling, the beam with articulated supports in A and D is taken into account, subject to 2 forces of 83.44 N, located symmetrically, the segment BC represents the finite rigidity specimen (see Figure 3), the modeling is carried out through the RISA 2D program. The moment obtained in the center of the specimen is M = -16688 N*mm (see Figure 4).

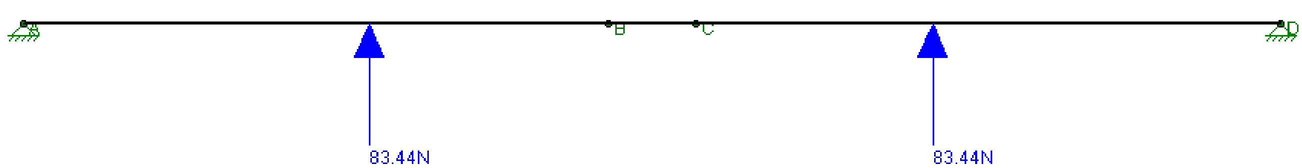

Figure 3. Beam with two articulated supports in A and D.

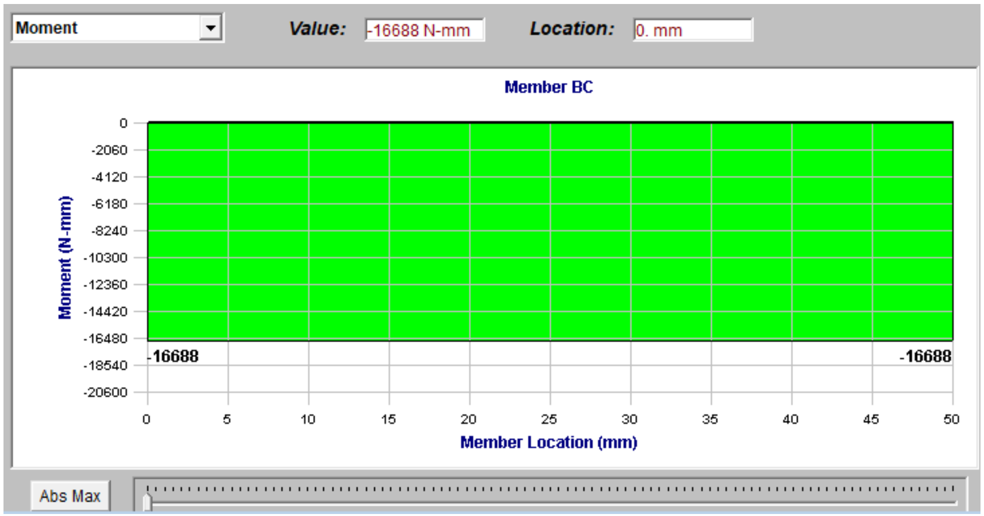

Figure 4. Results of the bending moment for the first proposed modeling.

3.2. Modeling 2.

In this modeling, it is considered that the connection between the machine shaft and the motor flange is a fixed support, there is a beam with two joints in A and D plus a fixed support in E (see Figure 5), the Modeling is carried out using the RISA 2D program. The moment that is obtained in the center of the BC is M = -583.229 N*mm (see Figure 6).

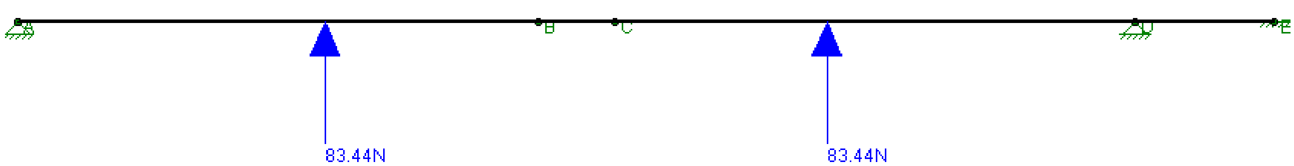

Figure 5. Beam with two articulated supports in A and D, fixed point in E.

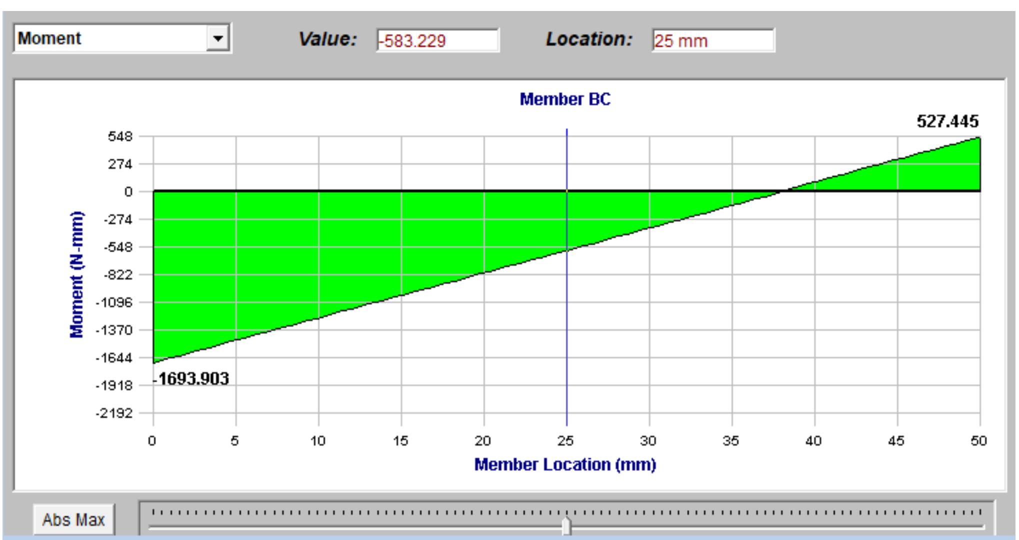

Figure 6. Results of the bending moment for the second proposed modeling.

3.3. Modeling 3.

In this modeling, a beam is considered with two joints in A and D, additional an spring-type support with a stiffness constant K=2KN/mm in point E (see Figure 7), the modeling is carried out using the RISA 2D program. The moment that is obtained for these conditions is M=-2770.054 N*mm (see Figure 8), it should be noted that at point E, there is a physically shaped flange and it has been modeled in the program as if it were a spring.

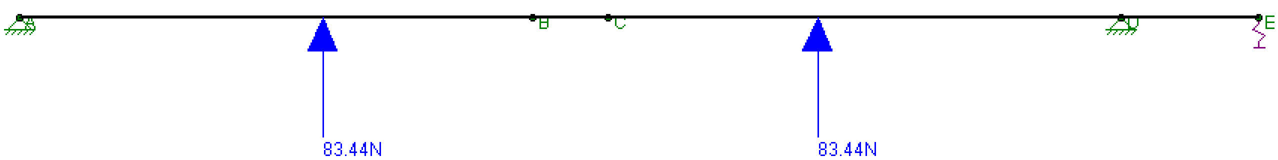

Figure 7. Beam with two articulated supports in A and D with spring type support in point E.

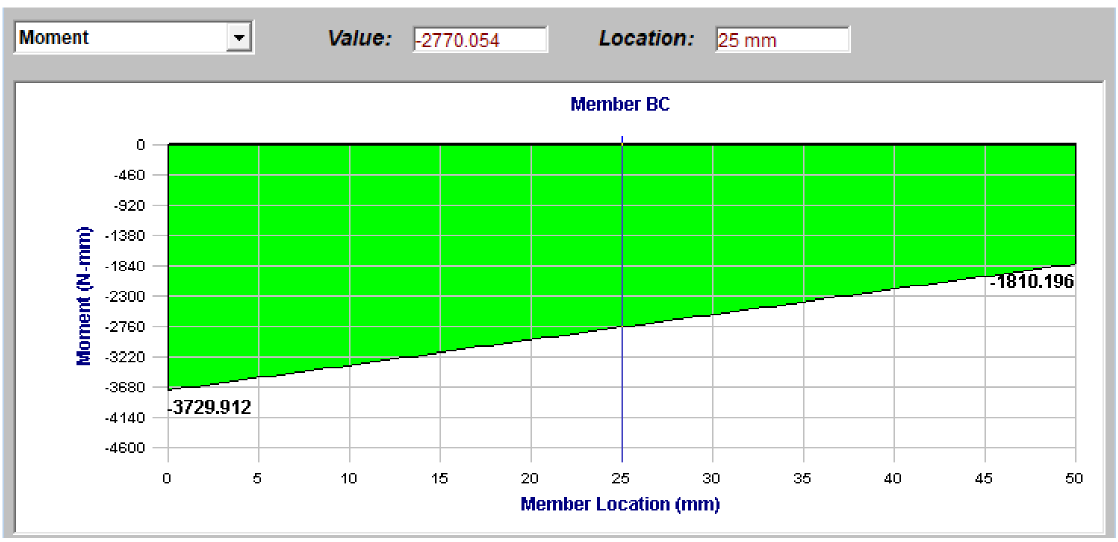

Figure 8. Results of the bending moment for the third proposed modeling.

3.4. Modeling 4.

For this modeling, a change in the design of the fatigue machine is proposed, which consisted of changing the flange that connects the motor and the shaft for a connection of two cardan type supports, this is represented by the IJ points and a joint type pin support in point K (see Figure 9), the modeling is carried out using the RISA 2D program. The moment obtained is M=-1652.761 N*mm (see Figure 10)

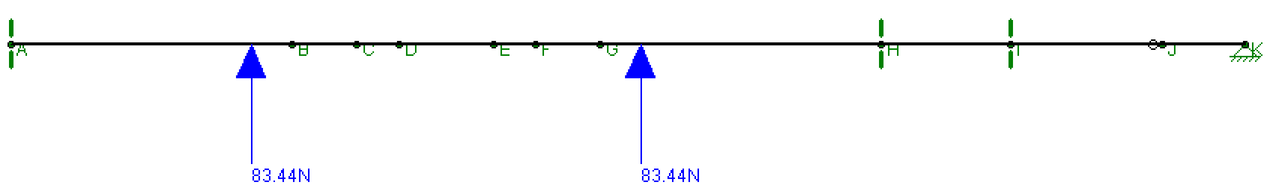

Figure 9. Beam with two articulated supports in A and D, cardan type supports in H and J, pin type support in E.

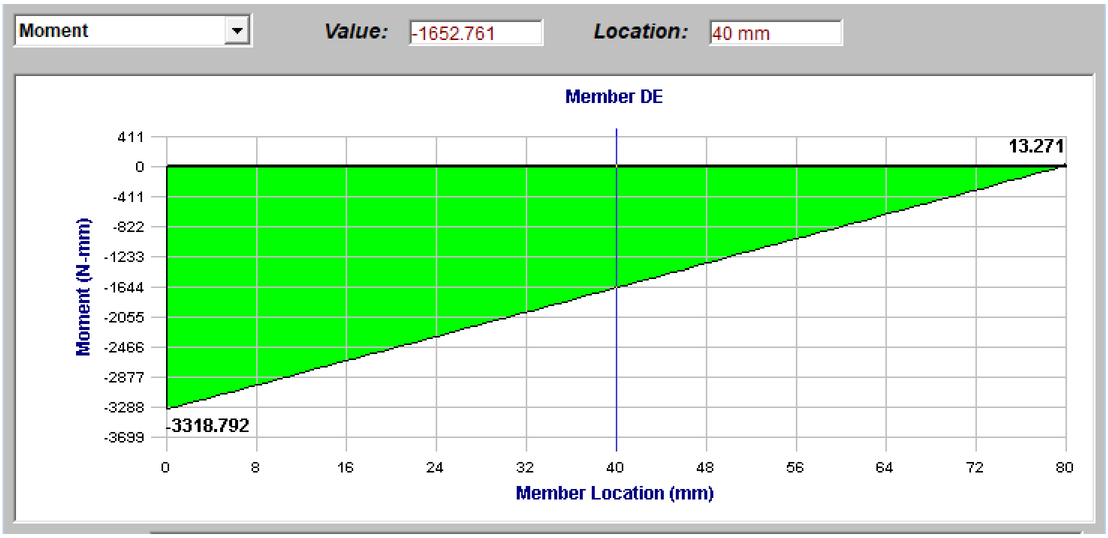

Figure 10. Results of the bending moment for the fourth proposed modeling.

3.5. Modeling 5.

Finally, the specimen BC is considered as a beam with flexural rigidity of a finite value and the two axes that hold it with infinite rigidity, subject to moments concentrated in the fixed points at B and C (see Figure 11), those are produced by a rotation in one of the extremes.

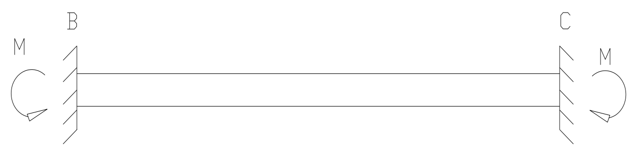

Figure 11. Beam with finite rigidity with concentrated moments.

To obtain the measured unit strain ϵ=297x10-6 from the simulation, we have a moment M= 3 N*m= 3000 N*mm for a unit strain ϵ=291x10-6 (see Figure 12).

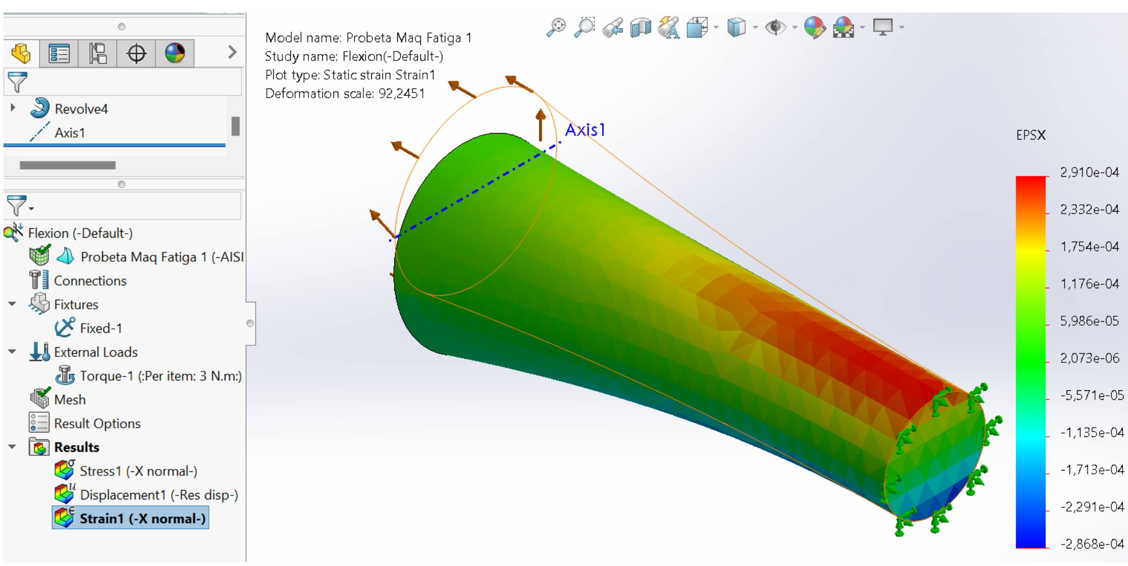

Figure 12. Unit strain applied a bending moment of 3000 N*mm.

From the previous analysis, results are summarized in Table 3.

Table 3. Error in percentage calculated for each simulation.

DESCRIPTION	MODELING	M (N*mm)	Error (%)
Two Joints	1	-16.688,00	487,07
Two joints plus embedment	2	-583,23	79,48
Two joints plus a spring type support	3	-2.770,05	2,55
Two joints plus two Cardan type supports	4	-1.652,76	41,86
Two embeddings with rotation	5	-3.000,00	5,54
Experimental Measurement		-2842,6	0,00

4. DISCUSSION

To enter into discussion about the most appropriate modeling, it is estimated that a confidence level greater than 95% is acceptable, so measurement errors of between 3% and 6% can be accepted.

Models 1, 2 and 4 exceed 6% error, so they are discarded as possible solutions to the problem posed. Model number one should be the modeling that fit in the best way but it is the one that is furthest from the expected value, which is why this study is presented.

Models 3 and 5 are the ones that are closest to the experimental model, both have acceptable values.

In the third modeling, it can be highlighted that for the RISA 2D program the flange motor assembly (support E of the modeling) can be replaced by a spring-type support whose constant is determined through experimental measurement, it is determined that this replacement can be carried out with a spring because the flange has rubber parts that absorb vibrations, when modeling, satisfactory results are obtained that prove the proposed theory.

The bars that act as axes, whose inertia is 10039.4 N*m^2, are considered as a model of infinite rigidity, and this value is 266.6 times the rigidity of the cross section in the test specimen.

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[2] José Pérez, 1ra edición, mayo 2023, ISBN: 978-9942-44-068-6 REGISTRO DEL INSTITUTO ECUATORIANO DE PROPIEDAD INTELECTUAL.

DOI: https://doi.org/10.15379/ijmst.v11i1.3574