Fuzzy Economic Order Quantity (EOQ) Model Through Fractional Backorder

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Abstract - This paper has analyzed fuzzy EOQ model through fractional back order for inventory system. Here, under function principle, fuzzy entire behavior cost of the model is calculated. The ideal EOQ is derived through median rule. This approach can be very helpful when the precise data of Fuzzy variables is unavailable. In this paper for recommended fuzzy EOQ model, the ideal resolution for the proposed fuzzy EOQ model is slightly greater than the economic order quantity (EOQ) in crisp value because of the scarcity of data. Partial backorder of fuzzy system, Setup and interrelated mechanism for implementation of fuzzy set for inventory system with partial backorder, A numerical example and conclusion are described in different sections.

Keywords - Economic order quantity; Median rule; Function principle; Fractional backorder

I. INTRODUCTION

Now a day's inventory plays a significant role in the supply chain management and also it makes up an important part of the entire industrial firm. Due to this cause, the success of the inventory organization disturbs the productivity of a firm. A public delinquent in inventory system is describing the importance of EOQ to decrease yearly inventory rate. The basic principle of Economic order quantity is presented by Haris [3].

$$\text{Economic Order Quantity } = \left(\frac{2RC}{H}\right)^{\frac{1}{2}} \quad \cdots\cdots\cdots \quad (1)$$

Where R indicates yearly demand; C indicates cost of ordering and H indicates cost of yearly stock.
Standard inventory models implement deterministic constraints. Though, in the routine circumstance there are numerous hesitations which must be reflected in implementation of inventory models. Maximum studies on Economic order quantity modelling habit probabilistic attitude to cope with insecurity. Economic order quantity model have convinced probability distribution, due to that model adopts tentative costs of carrying as well as holding. Though, numerous inventory constraints hesitations do not have any kind of expenditures evidence. In order to handle with such kind of situation, fuzzy Economic order quantity model is used. Also to address these shortcomings, we propose a Fuzzy model in order to signify the ambiguity as probability distribution.
The systematic verification, distribution and management information are necessary in order to apply Economic order quantity model. There are numerous revisions on fuzzy Economic order quantity modelling. C. Kao and W.K. Hsu suggested everyday demand of fuzzy model for a unique-period implemented through inventory model [1]. H. Lee and J. Yao suggested a fuzzy inventory structure by avoiding backorder structure [4]. J. Yao and J. Chiang focuses on centroid distance to analyze entire fuzzy rate and packing fuzzy rate in inventory without backorder [6]. Besides these several studies on a fuzzy Economic order quantity with backorder. S. Chen and C. Wang focuses on the fuzzy norm in order to modify fuzzy procedure for backorder fuzzy inventory [13]. They mainly focuses on fuzzy imperative cost, fuzzy demand, fuzzy backorder cost and fuzzy inventory cost, in their paper. H. Lee and J. Yao focuses on trilateral fuzzy digit to symbolize a fuzzy number $\tilde{Q}$ [4]. They used Centroid technique to defuzzify $\tilde{Q}$. During that year, H. Lee and J. Yao settled their model for backorder system [4]. Also trapezoidal participation function model was recommended by J. Yao and H. Lee [7]. Fuzzy constraints for inventory system through backorders explained by N. Kazemi and E. Ehsani [11]. An investigative explanation for a fuzzy EOQ proposed by K. Björk [8]. Trapezoidal fuzzy integer for economic order quantity with backorder applied by E. Bulancak and N. Kirkavak [2].
In this research paper try to extend the present fuzzy economic order quantity model to reflect incomplete backordering. For that mainly Function principle is used in EOQ fuzzy procedure. In addition, constraint uncertainty is reflected for demand rate (C), yearly request (R), allotment cost (H), damage cost (L) and backorder cost (K). The rest of the paper is planned in subsequent sections as follow: Crisp value Inventory Model implementation through partial backorder of fuzzy system is presented in section II; Setup and interrelated mechanism for implementation of fuzzy set for inventory system with partial backorder are describe in section III; A numerical example and conclusion are described respectively in section IV and V.

II. CRISP VALUE INVENTORY MODEL THROUGH PARTIAL BACKORDER

The systematic crisp value Economic order quantity model with partial backorder is reviewed in this section. Following Figure 1 describes the fractional backorder inventory.

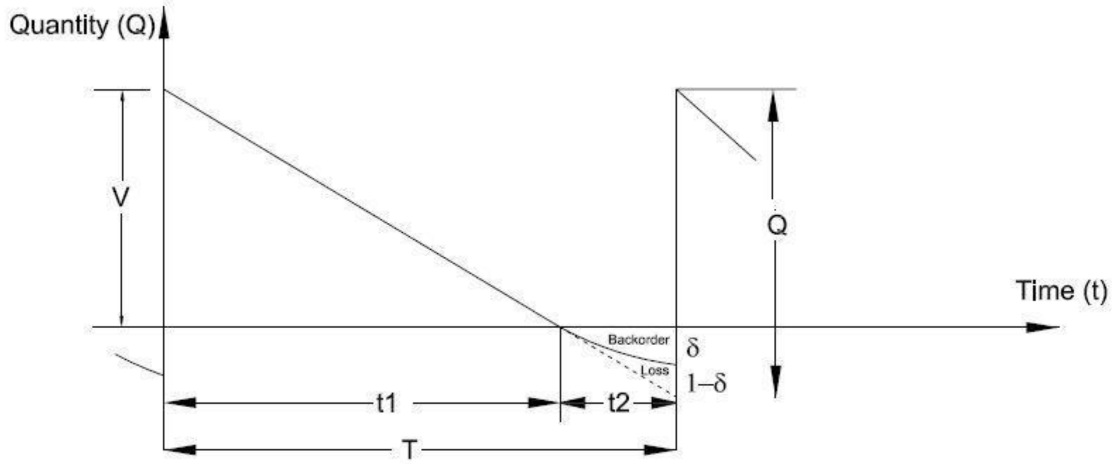

Figure 1. Fractional backorder inventory

Here we consider following Assumptions: Stocks are replaced immediately, Partial backorder is permissible and Single product is assumed.
Following notations are used for crisp value inventory model:
Total relevant cost :TRC, Reduction period : t1, Backorder period : t2, Yearly demand :R, Extreme inventory :V, Ideal order quantity :Q, Order rate : C, Allotment cost : , Backordering rate : K, Damage sale cost :L and Backorder portion :δ.
In this paper, derived the total relevant cost for fuzzy economic order quantity model from allotment cost, order cost, damage sale cost and backorder cost. Here portion of the backorder item is symbolized by δ. Also there are two stages of inventory cycle in the model, of which first one is depletion period (t1) and second is backorder period (t2). The values depletion period (t1) and backorder period (t2) are defined as under:

$$t_1 = T\left(\frac{V}{Q}\right) \quad \cdots\cdots\cdots\cdots \quad (2)$$

$$t_2 = T\left(\frac{Q-V}{Q}\right) \quad \cdots\cdots\cdots\cdots \quad (3)$$

Here the order cost, allotment cost, backorder cost and damage sale cost are formulated as under:

1. Order cost:

$$\text{Order cost} = \frac{C\,R}{Q} \quad \cdots\cdots\cdots\cdots \quad (4)$$

2. Allotment cost:

$$\text{Allotment cost} = \frac{H\,V^2}{2\,Q} \quad \cdots\cdots\cdots\cdots \quad (5)$$

3. Backorder cost:

$$\text{Backorder cost} = \frac{K\,\delta\,(Q-V)^2}{2\,Q} \quad \cdots\cdots\cdots\cdots \quad (6)$$

4. Damage cost:

$$\text{Damage cost} = \frac{L(1-\delta)(Q-V)^2}{2\,Q} \quad \cdots\cdots\cdots\cdots \quad (7)$$

The total relevance cost is the addition of above all order cost, allotment cost, backorder cost and damage cost.

$$\therefore\ \text{Total relevance cost} = \frac{C\,R}{Q} + \frac{H\,V^2}{2\,Q} + \frac{K\,\delta\,(Q-V)^2}{2\,Q} + \frac{L(1-\delta)(Q-V)^2}{2\,Q} \quad \cdots\cdots\cdots\cdots \quad (8)$$

In order to occurs Optimize solution:

$$\frac{\partial TRC}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial TRC}{\partial V} = 0$$

Solving above two conditions we get:

$$Q = \pm \sqrt{\frac{2CR\{H + \delta K + (1-\delta)L\}}{H\{\delta K + (1-\delta)L\}}} \quad \cdots\cdots\cdots\cdots \quad (9)$$

As optimum order quantity Q ≥ 0, only the positive value of Q is considered.

$$V = + \sqrt{\frac{2CR\{H + \delta K + (1-\delta)L\}}{H\{\delta K + (1-\delta)L\}}} \quad \cdots\cdots\cdots\cdots \quad (10)$$

If fraction of backorder δ=1; equation (9) and (10) reduces to

$$Q = \sqrt{\frac{2CR\{H+K\}}{H\,K}} \quad \cdots\cdots\cdots\cdots \quad (11)$$

$$V = \sqrt{\frac{2CR\{H+K\}}{H\,K}} \quad \cdots\cdots\cdots\cdots \quad (12)$$

Above results in equation (11) and (12) in a completely backordered model. As per suggested by R.J.Tersine [12]; the equations (11) and (12) are too much similar to economic order quantity model for fully backorder system. Also if fraction of backorder $\delta = 0$; equation (9) and (10) reduces to

$$Q = \sqrt{\frac{2CR\{H+L\}}{H\,L}} \quad \cdots\cdots\cdots\cdots \quad (13)$$

$$V = \sqrt{\frac{2CRL}{H\,(H+L)}} \quad \cdots\cdots\cdots\cdots \quad (14)$$

Above equation (13) and (14) result in a comprehensive lost sale model.

III. PARTIAL BACKORDER INVENTORY MODEL FOR FUZZY MODEL

Following notations are used for fuzzy model:
fuzzy model Total relevant cost : $\widetilde{TRC}$, allotment cost of fuzzy number : $\widetilde{H}$, cost of ordering for fuzzy number : $\tilde{C}$, yearly demand for fuzzy number: $\tilde{R}$, backordering cost for fuzzy number : $\widetilde{K}$, damage sale cost of fuzzy number :$\widetilde{L}$, multiplication of Fuzzy number : ⊗, division of fuzzy number : Ø , summation of fuzzy number : ⊕ ,fuzzy model ideal order quantity : $\widetilde{Q}$, fuzzy model maximum inventory : $\widetilde{V}$, the unit rate of participation function for x :$\mu_{\tilde{A}}(x)$, participation function value : w
Here the entire significance rate using fuzzy model from equation (8) through fuzzy operation, is obtained as under:

$$\text{Total relevance cost} = \frac{\tilde{C} \otimes \tilde{R}}{\tilde{Q}} + \frac{\widetilde{H} \otimes \widetilde{V^2}}{2V\tilde{Q}} + \frac{\widetilde{K} \otimes \delta\left(\tilde{Q}-\tilde{V}\right)^2}{2\,\tilde{Q}} + \frac{\tilde{L} \otimes (1-\delta)\left(\tilde{Q}-\tilde{V}\right)^2}{2\,\tilde{Q}} \quad \cdots\cdots\cdots\cdots \quad (15)$$

In this paper, concept of trapezoidal rule is also used for the function of fuzzy participation. Here the trapezoidal association function for variable$A_{\sim}$ is defined in (16) as under:

$$\mu_{\tilde{A}}(x) = \begin{cases} w(x-a_1)\,/\,(a_2-a_1)\,;\, a_1 \le x \le a_2 \\ \quad w \quad ; \quad a_2 \le x \le a_3 \\ w(x-a_4)\,/\,(a_3-a_4)\,;\, a_3 \le x \le a_4 \\ \quad 0 \quad ; \quad otherwise \end{cases} \quad \cdots\cdots\cdots\cdots \quad (16)$$

Where 0 < w < 1

For fuzzy operation, function principle is used. Here the membership function $\tilde{A}$ of trapezoidal rule for fuzzy participation is represented as $(a_1, a_2, a_3, a_4; w)$ where $a_1 < a_2 < a_3 < a_4$. For the two trapezoidal membership function $\tilde{A}$and $\tilde{B}$, using the function principle, their fuzzy calculation operations for addition and multiplication are defined in following equations (17) and (18) as under:

$$\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4; w) \quad \cdots\cdots\cdots\cdots \quad (17)$$

$$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4; w) \quad \cdots\cdots\cdots\cdots \quad (18)$$

Where w is the smallest value between w1 and w2. The fuzzy arithmetic operation are explained in Figure 2 as under.

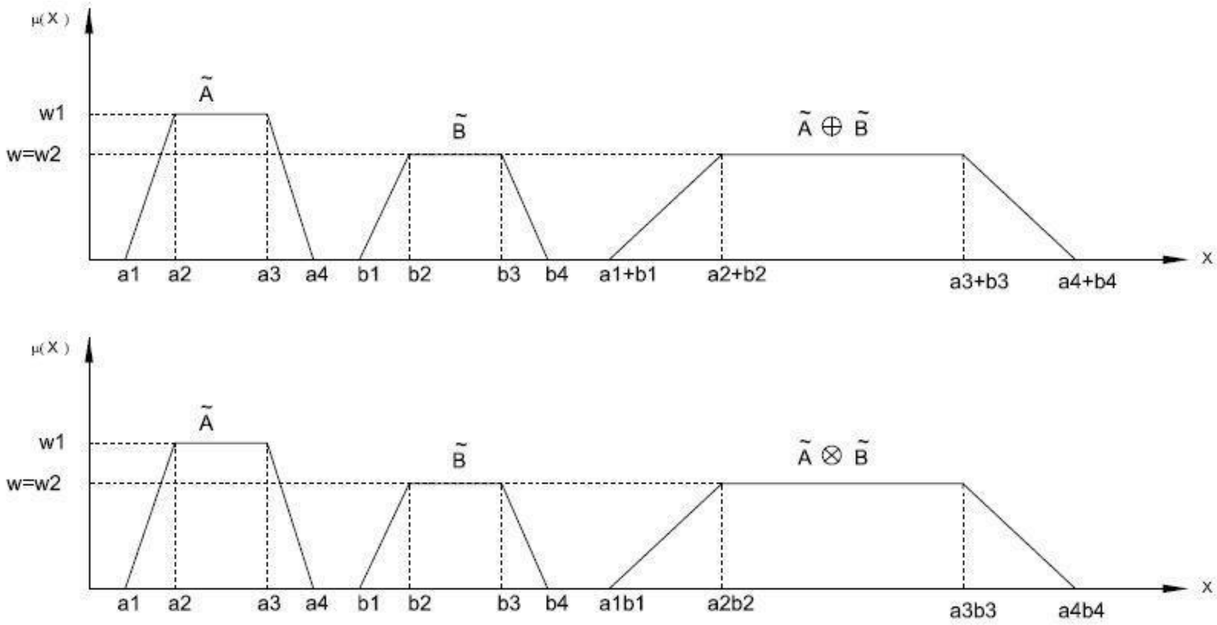

Figure 2. Fuzzy arithmetic operation

With the intention of fuzzy variable defuzzify ($\tilde{A}$), used the median rule suggested by K.S.Park [14]. The median of a_m for fuzzy set $\tilde{A}$ is obtained from:

$$\frac{2\,a_m - a_1 - a_2}{2} = \frac{a_3 + a_4 - 2a_m}{2} \quad \text{where} \quad a_m = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

In this study, considering the only case for which $a_1 < a_2 < a_3 < a_4$.

The entire significance cost depends on the median rule, can be defined as under:

$$\widetilde{\mathrm{TRC}} = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{c_i\, r_i}{q_i} + \frac{h_i\, v_i^{\;2}}{2\, q_i} + \frac{k_i\, \delta\, (q_i - v_i)^2}{2\, q_i} + \frac{l_i (1-\delta)(q_i - v_i)^2}{2\, q_i} \right) \quad \cdots\cdots\cdots\cdots \quad (19)$$

Now differentiate equation (19) partially with respect to $\widetilde{TRC}$,

$$\frac{\partial \widetilde{\mathrm{TRC}}}{\partial Q} = \frac{1}{4} \sum_{i=1}^{4} \left\{ \frac{-c_i\, r_i}{q_i^2} - \frac{h_i\, v_i^{\;2}}{2\, q_i^2} + \frac{k_i\, \delta}{2} \left(1 - \frac{v_i^2}{q_i^2} \right) + \frac{l_i (1-\delta)}{2} \left(1 - \frac{v_i^2}{q_i^2} \right) \right\}$$

And

$$\frac{\partial \widetilde{\mathrm{TRC}}}{\partial V} = \frac{1}{4} \sum_{i=1}^{4} \left\{ \frac{h_i\, v_i}{q_i^2} + \frac{k_i\, \delta}{2\, q_i} (2\, v_i - 2\, q_i) + \frac{l_i (1-\delta)}{2\, q_i} (2\, v_i - 2\, q_i) \right\}$$

In order to find Optimum solution, use the following conditions:

$$\frac{\partial \widetilde{\mathrm{TRC}}}{\partial Q} = 0 \quad \text{and} \quad \frac{\partial \widetilde{\mathrm{TRC}}}{\partial V} = 0$$

Hence using the above conditions, the obtaining solutions are:

$$\tilde{Q} = \sqrt{\sum_{i=1}^{4} \left(\frac{2\, c_i\, r_i}{h_i} \right)} \sqrt{\frac{\sum_{i=1}^{4} h_i + \sum_{i=1}^{4} k_i \delta + \sum_{i=1}^{4} l_i (1-\delta)}{\sum_{i=1}^{4} k_i \delta + \sum_{i=1}^{4} l_i (1-\delta)}} \quad \cdots\cdots\cdots\cdots \quad (20)$$

$$\tilde{V} = \sqrt{\sum_{i=1}^{4} \left(\frac{2\, c_i\, r_i}{h_i} \right)} \sqrt{\frac{\sum_{i=1}^{4} k_i \delta + \sum_{i=1}^{4} l_i (1-\delta)}{\sum_{i=1}^{4} h_i + \sum_{i=1}^{4} k_i \delta + \sum_{i=1}^{4} l_i (1-\delta)}} \quad \cdots\cdots\cdots\cdots \quad (21)$$

When the fuzzy involvement functions contains a single value, such that $\tilde{C}\ (c_1 = c_2 = c_3 = c_4) = C$, the values of $\tilde{Q}$, $\tilde{V}$and Total relevant cost ($\widetilde{\text{TRC}}$) are as under:

$$\tilde{Q} = \sqrt{\frac{2CR\ \{H + \delta K + (1-\delta)L\}}{H\ \{\delta K + (1-\delta)L\}}}$$

$$\tilde{V} = \sqrt{\frac{2CR\ \{\ \delta K + (1-\delta)L\}}{H\ \{H + \delta K + (1-\delta)L\}}}$$

$$\widetilde{\text{TRC}} = \frac{C\ R}{Q} + \frac{H\ V^2}{2\ Q} + \frac{K\ \delta\ (Q-V)^2}{2\ Q} + \frac{L(1-\delta)(Q-V)^2}{2\ Q} \qquad \cdots\cdots\cdots\cdots\ (22)$$

Here above equation (22) is the same as previous equation (8) ; that means we can deliberate fuzzy membership as crisp value when all they consists of a single value, hence we say that $\widetilde{TRC}$ is the equivalent as TRC .

IV. STATISTICAL EXAMPLE

Fuzzy economic order quantity model is a simple unsupervised learning method which can be used to obtaining optimal solution through function principle. Now a day's main requirements of corporation is to estimate the total relevance cost and economic order quantity for a product in a minimization way regarding to production cost. Though, the corporation doesn't have any kind of thorough information regarding the order cost, upcoming yearly demand, allotment cost, damage sale cost and backorder cost. The establishment expected that the range of yearly demand $\tilde{R}$is expected between 3000 and 5000 units per year. The allotment cost $\widetilde{H}$ is between the ranges of \$1 to \$4 per unit. Also the total order cost $\tilde{C}$is expected between \$10 and \$20 per order. Damage sale cost $\tilde{L}$is expected to be around \$7 to \$17 per unit. The backorder cost $\widetilde{K}$is predictable to be around \$3 to \$9 per product. Also here the value for the portion of backorder δ is 50%.
According to the suggested approach, to solve this problem, in order to characterize the ambiguity of data, consistent trapezoidal membership functions are used. Here the maximum probable value is roughly around the mean of the range value. The maximum participation value is 1. For each probable cost, the association tasks are as under:

$$\tilde{R} = (3000\ , 3500\ , 4500\ , 5000)$$
$$\tilde{C} = (10\ , 15\ , 25\ , 20)$$
$$\widetilde{H} = (1.5\ , 2\ , 3\ , 3.5)$$
$$\widetilde{K} = (3, 4\ , 7\ , 9)$$
$$\tilde{L} = (7, 10\ , 14\ , 17)$$

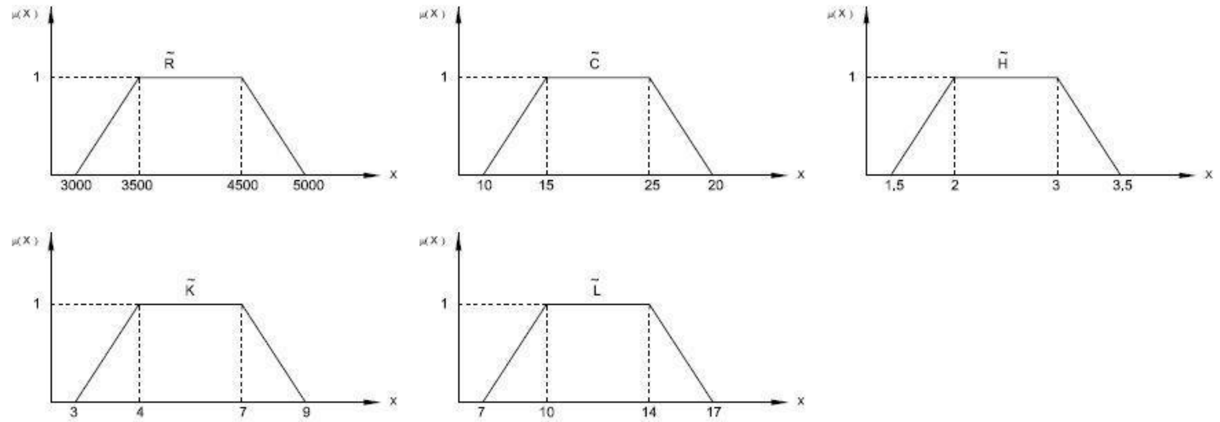

Figure 3. Fuzzy variables participation function

The graph, which illustrated in above figure 3, is represents the fuzzy variables participation function for different probable cost.

Calculating$\widetilde{TRC}$:
Step 1: Calculate $\tilde{Q}$ and $\tilde{V}$ using (20) and (21);
$Q^{\sim} = 385.43$ and $\tilde{V} = 274.78$

Step 2: Assigning $\widetilde{Q}$ and $\tilde{V}$to (19);

$\text{Total relevant cost} = 1570.65$

While the fuzzy variables have a distinct value, $\tilde{C}\ (c_1 = c_2 = c_3 = c_4) = C$, it behaves as a crisp value. If the yearly demand, allotment cost, demand cost, damage sale cost and backorder cost values are taken from their average values, their respective values are:
$R^{\sim} = 4500$, $\tilde{C} = 15$, $\widetilde{H} = 2.5$, $\widetilde{K} = 6$ and $\tilde{L} = 12$. In this case the fuzzy association function becomes as under and whose calculations are also denoted by graphs in figure 4 as under:

$$\tilde{R}\ (r_1 = r_2 = r_3 = r_4) = 4500$$
$$\widetilde{C}\ (c_1 = c_2 = c_3 = c_4) = 15$$
$$\widetilde{H}\ (h_1 = h_2 = h_3 = h_4) = 2.5$$
$$\widetilde{K}\ (k_1 = k_2 = k_3 = k_4) = 6$$
$$\tilde{L}\ (l_1 = l_2 = l_3 = l_4) = 12$$

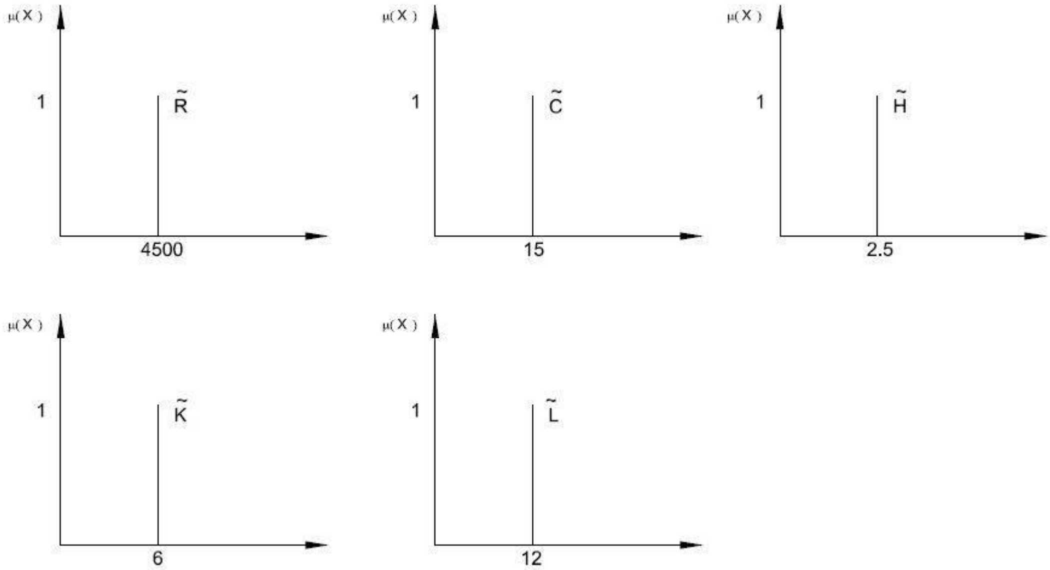

Figure 4. Fuzzy variables for single value

The graph, which illustrated in above figure 4, is represents the fuzzy variables participation function for single value.
The $\widetilde{\text{TRC}}$ with convinced average value is as under:
Step 1: Calculate $\tilde{Q}$ and $\tilde{V}$ from equation (20) and (21);
$\tilde{Q} = 385.22$
$\tilde{V} = 330.15$
Step 2: Putting $Q^{\sim}$ and $\tilde{V}$ in equation (19) we obtain;
$\widetilde{TRC}$= 1062.75

We can evaluate total relevant cost for actual rate, by using equation (8).

Here the total relevant cost (TRC) with statistics is as under:
Step 1: Evaluate Q and Vusing equations (9) and (10) , we obtain the values ;
$Q = 385.22$
$V = 330.15$
Step 2: Putting Q and $V$to (19) , we obtain value of total relevant cost ;
$TRC = 1045.18$

The above calculation indicates that when there is a lack in information, the calculated value of $\widetilde{TRC}$ is greater than TRC. This is a truthful consequence as the fuzzy $\widetilde{TRC}$reflects the ambiguity due to a deficiency of evidence. This can be used to evaluate the possibility cost because of lack of constraints information. In addition numerous other trials were also conducted to determine the effectiveness of the system under fluctuating circumstances, which improves the accuracy of the fuzzy economic order quantity model.

V. CONCLUSION

Now a days in recent creation where revolutions of innovative technologies are expected day by day in order to fulfil the client's requirement and emergency demand, industries or institute has continuously watch on the universal revolutions, associations and efficiency which is too much compulsory for the association in order to compete with other establishments. As a part of these, Fuzzy model plays an important role in inventory theory. In the proposed work, a mathematical representation of fuzzy Economic order quantity model is presented for inventory through partial backorder technique. Here the ratio for the portion of backorder δ is used to denote the proportion of backordering. When, $\delta = 1$, one has a comprehensive backordering model. Alternatively, when $\delta = 0$, all backorder will be missing sale.

The investigations carried out in this paper proves that the fuzzy Economic Order Quantity Model methods is capable of estimating the Partial Backorder value for inventory systems. Fuzzy variables are appropriate when the precise information is not accessible. Due to lacking of information, fuzzy model is used in to model improbability. In this paper, median rule is used to discover the fuzzy variable minimization however a function principle is apply in the fuzzy operation. The proposed approach is capable of handling uncertain, complex and non-linear data whereas the fuzzy model will result in a higher cost. This is essential as we deliberate whether to invest a supplementary cost on additional information. This indicates that the estimation of the fuzzy data is associated with severe environmental complexity and nonlinearity. Moreover, the proposed technique can be used by industrialist in order to minimize inventory cost in any industry.

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