# A Model Study on the Effects of Π-Mesons on Thermodynamics of Strongly Interacting Systems

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**Abstract:** Properties of strongly interacting matter are quite well-described by the Polyakov loop enhanced Nambu-Jona—Lasinio (PNJL) model as it incorporates two important symmetries. However, discrepancies in low temperature domain remain unless hadronic effects are entwined. Here we have tried to observe the effect of lowest lying hadrons i.e. π-mesons (pions) considering their medium-dependent masses for zero chemical potential. The thermodynamic quantities now show satisfactory matches with existing lattice QCD and HRG results over the finite temperature window of interest.

Keywords: PNJL model, π-meson, strongly interacting system, Thermodynamics.

## 1. INTRODUCTION

Relativistic heavy-ion collision experiments and associated studies are of prime interests for researchers all over the World. In connection to the phase transition properties of such exotic matter produced in these experiments mimicking the early stages of the Universe, the cross-over temperatures as reported by Wuppertal-Budapest (WuB) and HotQCD collaborations are around 150 and 155 MeV respectively [1,2]. Parallelly, various QCD-inspired models are built using one or more symmetry criteria. One of them is Nambu-Jona-Lasinio model which takes care of breaking of the chiral symmetry and its restoration in QCD quite well. The gluonic effect and thereby the effect of confinement is included on addition of the Polyakov loop. This effectively acts as a temporal background gluon field. Thus, the Polyakov loop enhanced Nambu-Jona-Lasinio (PNJL) model encapsulates both chiral and deconfinement scenario under a single framework. We then seek to study various thermodynamic quantities under mean field approximation.

As Lattice QCD progressed to reach the continuum limit, the results became distinctively different from earlier ones. To stay concurrent, modifications in PNJL model have been done [3]. However, quantitative discrepancies were still observed in the low temperature domain, primary cause to which can be identified as the absence of hadronic degrees of freedom. Different attempts have been made in this direction by different groups as in [4, 5, 6]. Combining HRG model to PNJL using suitable switching function has also been tried previously [7]. However, recognizing that the low-lying hadronic states would be mostly contributing to pressure and hence other thermodynamic variables, here we attempt to see their effects systematically.

The medium-dependent mass of π-meson (pion) should act as a natural switching function to provide appropriate physics in the low temperature domain. The issue of suppression of contributions from constituent quark masses by the confinement feature of the Polyakov loop should thus be bypassed. We present here the results for zero chemical potentials, as primarily we aim to come in proximity with lattice QCD. The framework can then be modified for finite chemical potential (μ), which we plan to present elsewhere.

We briefly describe the PNJL model in the next section, followed by the calculation of pion mass. Finally, we present the results with adequate discussions. Lastly, we conclude.

## 2. MODEL FRAMEWORK

### 2.1. PNJL Model

Let us now discuss the framework of PNJL model employed here [3], which is structured by suitably adding P-loop to the NJL model [8,9,10]. The NJL part takes care of the chiral properties whereas the P-loop part essentially works towards confinement. Previously there were various studies using PNJL model with 2 as well as 2+1 flavors [8, 11-20]. Here we have considered 2+1-flavor PNJL model with 6-quark interactions. The thermodynamic potential takes the form [3],

$$\Omega(\varphi,\bar{\varphi},\sigma_f,T,\mu) = 2g_S\sum_{f=u,d,s}\sigma_f{}^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s - 6\sum_f\int_0^\Lambda \frac{d^3p}{(2\pi)^3}E_f\Theta(\Lambda-|\vec{p}|) - 2T\sum_f\int_0^\infty \frac{d^3p}{(2\pi)^3}\, ln\left[1+3\left(\varphi+\bar{\varphi}\, e^{-(E_f-\mu_f)/T}\right)e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}\right] - 2T\sum_f\int_0^\infty \frac{d^3p}{(2\pi)^3}\, ln\left[1+3\left(\bar{\varphi}+\varphi e^{-(E_f+\mu_f)/T}\right)e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}\right] + \mathcal{U}'(\varphi,\bar{\varphi},T) \qquad (1)$$

The first five terms in Eq.(1) are contributions from NJL model, modified suitably in presence of P-loop.

$$\sigma_f = \langle\overline{\Psi_f}\,\Psi_f\rangle$$

are the condensates for two light and one heavy quarks viz., f = u, d, s. $g_S$ and $g_D$ are the coupling coefficients for 4-quark and 6-quark type interactions, where the latter is responsible for explicit symmetry breaking of $U_A(1)$. $E_f$=$\sqrt{p^2+M^2}$ is the quasiparticle energy with $M_f$ being the associated mass of the corresponding meson,

$$M_f = m_f - 2g_S\sigma_f + \frac{g_D}{2}\,\sigma_{f+1}\sigma_{f+2} \qquad (2)$$

where, if f=u then f+1 =d and f+2=s and so-on in clockwise manner.

The third term in the RHS of (1) is the zero-point energy, whereas the fourth and fifth terms are effects from quarks and antiquarks at finite T & μ.

$\varphi = \frac{Tr_cL}{N_c}$ and, $\bar{\varphi} = \frac{Tr_cL^\dagger}{N_c}$ are the P-loop fields and its conjugate with the Polyakov loop,

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{1/T} d\tau\, A_4(\vec{x},\ \tau)\right]$$

$A_4$ being the temporal component of the gluon field acting in the background. The potential for $\varphi$ and $\bar{\varphi}$ [13, 21-24 ] is given by $\mathcal{U}'$ as,

$$\frac{\mathcal{U}'(\varphi,\bar{\varphi},T)}{T^4} = \frac{\mathcal{U}(\varphi,\bar{\varphi},T)}{T^4} - \kappa\ln[J(\varphi,\bar{\varphi})] \qquad (3)$$

$$\& \quad \frac{\mathcal{U}(\varphi,\bar{\varphi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\varphi}\varphi - \frac{b_3}{6}(\varphi^3+\bar{\varphi}^3) + \frac{b_4}{4}(\bar{\varphi}\varphi)^2 \qquad (4)$$

This $\mathcal{U}$ is therefore a Landau-Ginzburg type potential in accordance with the global Z(3) symmetry [9]. $b_2(T)$ is chosen as

$$b_2(T) = a_0 + a_1\exp\left(-a_2\,\frac{T}{T_0}\right)\frac{T_0}{T} \qquad (5)$$

where, $b_3$ and $b_4$ are constants. To take care of the transformation from P-loop to the traces, we have used $J[\varphi,\bar{\varphi}]$ as the Jacobian & κ is determined computationally acting as a dimensionless parameter. All the parameter values used in this framework are listed in Tables I and II. Previously a cross-over temperature ~ 160 MeV was reported in [3]. Results were in close agreement with LQCD data. However, discrepancies were observed in low temperature regime. As a solution, use of suitable switching function was proposed in [7]. Here however a different and a more natural scheme is used, considering the medium dependent mass of the lowest lying hadron viz. pion. The method is described in the next section.

**Table I : Parameters in the NJL model**

| $m_u$ (MeV) | $m_s$ (MeV) | $\Lambda$ (MeV) | $g_s\Lambda^2$ | $g_D\Lambda^5$ |
|---|---|---|---|---|
| 5.5 | 134.758 | 631.357 | 3.664 | 74.636 |

**Table II : Parameters for the Polyakov loop**

| $T_0$(MeV) | $a_0$ | $a_1$ | $a_2$ | $b_3$ | $b_4$ | κ |
|---|---|---|---|---|---|---|
| 175 | 6.75 | -9.0 | 0.25 | 0.805 | 7.555 | 0.1 |

### 2.2. Pions in PNJL model

As discussed, here we are trying to see the effect of lowest lying hadrons viz. pions to observe the effects on thermodynamics of strongly interacting matter specially in the low temperature domain. Methods of such rectifications have been tried before by concatenating HRG and PNJL results using suitable switching function [7]. Here the medium dependence of pion mass should serve the purpose more naturally and elegantly.

We consider the potential in Eq. (1) under mean-field approximation. The beyond mean-field calculations can be found in [25-27]. The next to leading order calculation in $\frac{1}{N_c}$ expansion considering forms of ring diagram lead to the mesonic (M) contributions in the potential, in a form like,

$$\delta\Omega_M = g_M \int \frac{d^3p}{(2\pi)^3} \int d\omega \left[\frac{\omega}{2} + T\,ln\left(1 - e^{-\frac{\omega}{T}}\right)\right] * \frac{1}{\pi}\, \frac{d\delta_M(\omega,\vec{p},T)}{d\omega} \tag{6}$$

$g_M$ here refers to internal mesonic degrees of freedom, $\delta_M(\omega, \vec{p}, T)$ being scattering phase shift of quark and anti-quark in M channel.

Temperature-dependent mesonic masses can be obtained from the pole condition,

$$1 - 2G_M\, \Pi_M\left(\omega = m_M, \vec{k} = 0\right) = 0 \tag{7}$$

$G_M$ here refers to the effective vertex factor. $\Pi_M(k^2)$, corresponding to particular mesonic channel, is one-loop polarization function given by the random phase approximation as,

$$\Pi_M(k^2) = \int \frac{d^4p}{(2\pi)^4}\, Tr[\Gamma_M S(p + \frac{k}{2})\Gamma_M S(p - \frac{k}{2})] \tag{8}$$

S(p) here refers to the quark propagator. $\delta\Omega_M$, the mesonic contribution to thermodynamic potential can be written as,

$$\delta\Omega_M = -\nu_M T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-\frac{E_p}{T}}\right) \tag{9}$$

$\nu_M$ here corresponds to statistical weight factor of corresponding species.

$$E_p = \sqrt{\vec{p}^2 + {m_{pole}}^2(T)} \tag{10}$$

where $m_{pole}$ is the mass of the meson, in this case corresponding to pionic mass, which is obtained by solving Eq.(7).

## 3. RESULTS

We intend to study various thermodynamical quantities. To start with, the thermodynamic potential (1) is minimized with respect to σ, $\varphi$ and $\bar{\varphi}$ and the mean fields are obtained henceforth. Inserting these into Eqs. (7) and (8), we get pion mass as function of temperature. Substituting the mean fields in Eq. (1) and the pole masses from Eq. (7), we obtain the value of pressure of pion from Eq. (9) and the total pressure by summing up Eqs. (1) and (9). The scaled pressure is shown in Fig. 1. The continuum extrapolated lattice QCD data, from both HotQCD as well as the Wuppertal-Budapest collaborations are plotted alongside. We observe that the pressure obtained using this modified version of PNJL model matches the PNJL and lattice QCD results for higher temperatures. It is interesting to see that at low temperatures, though results obtained using old version of PNJL model fails to match lattice data, our reframed modified PNJL model shows excellent parity with lattice results.

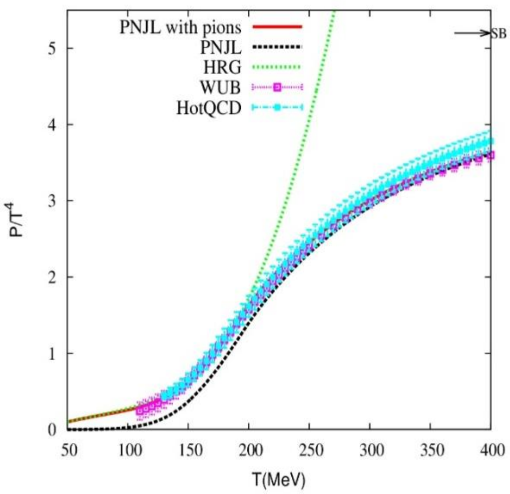


**Fig. 1 :** Pressure as a function of temperature

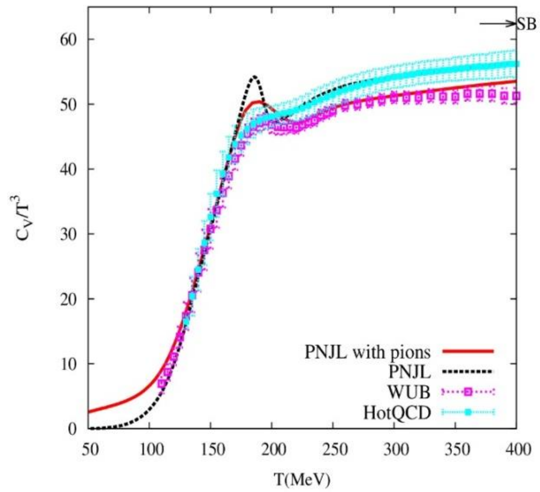


**Fig. 2:** Specific heat as a function of temperature

With this modified model incorporating pions, we now study the specific heat at fixed volume. $C_V$ is a very important observable, specifically since it is the second derivative of thermodynamic potential with respect to temperature. Its behavior is shown in Fig.2. It is enticing to see how the scaled specific heat shows a signature at the transition when computed using PNJL model and its revised version. This gives an exciting edge over lattice results.

Moving on, we portray the behavior of energy density with temperature in Fig 3. Energy density being first order derivative of pressure, is expected to show an order paramater-like behavior. It is delightful to observe the match of PNJL and lattice results again at high temperature domain only. The more interesting fact here is that the difference between PNJL and the modified structure using pions, becomes more evident and we see the PNJL results diverging away from lattice, thus re-establishing the importance and significance of including pions in the study.

In Fig. 4, we see the behavior of scaled entropy density with temperature. We compare results obtained from both the frameworks with lattice continuum results. After displaying the quantitative computations, we see that at higher temperatures, outcomes from both the models brilliantly match with lattice results. However, at lower temperatures, the picture is entirely different. Here, though PNJL model fails to match with lattice, our modified version with pions brilliantly falls in line with lattice results. Though we found a similar match using switching function too, however there, tuning of parameters needed to be done, unlike here, where temperature-dependent pion mass act as natural switching function.

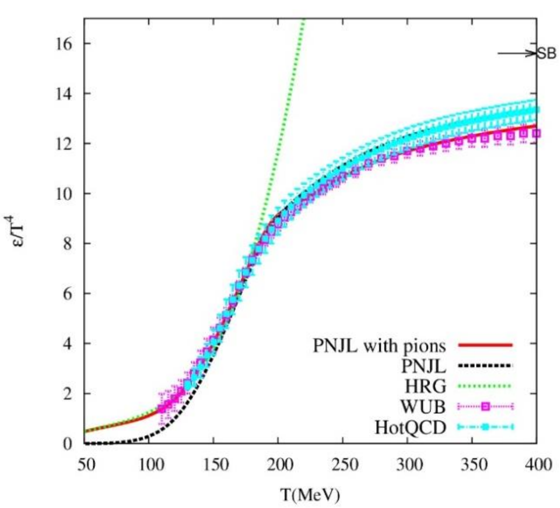


**Fig. 3:** Energy density as a function of temperature

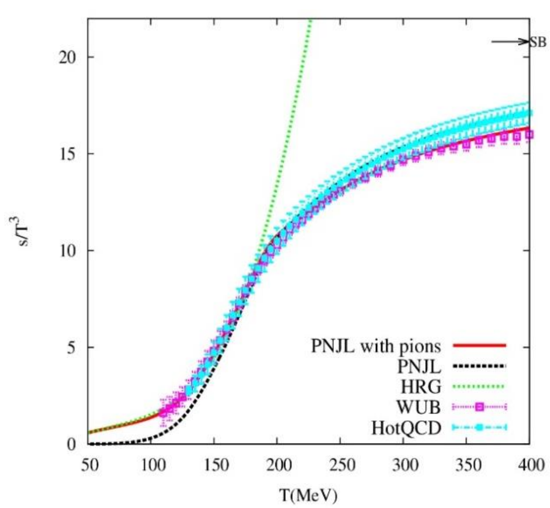


**Fig. 4:** Entropy density as a function of temperature

Another quantity which is very important for critical assessment of any theory is the trace of the energy-momentum tensor also called the Trace anomaly, which is defined in the following manner,

$$\Theta_{\mu\mu} = \epsilon - 3P$$

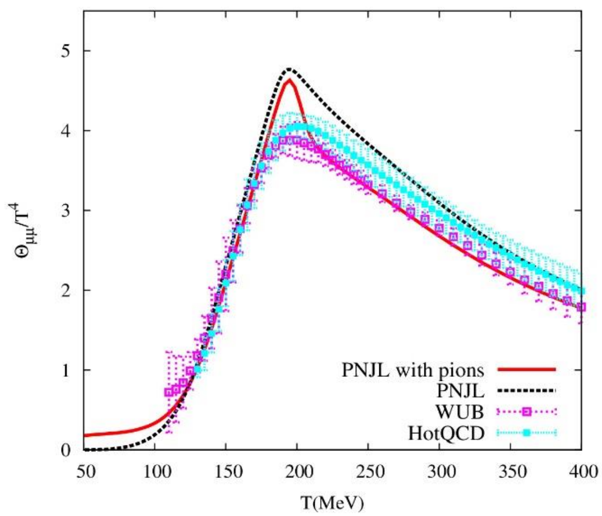


**Fig. 5:** Trace anomaly as a function of temperature

It is expected that for a conformal theory, this quantity will vanish. However, when due to any quantum interaction, some scale is introduced in the theory like cross-over or transition temperature, then value of this quantity diverges from zero. This proves the sensitivity of this quantity and its importance to characterize any theory. Here also, we see that though PNJL results show disparity with lattice over almost the whole temperature regime, however, modified PNJL with pions shows close proximity with lattice. The gap between PNJL and lattice is thus brilliantly overcome on incorporation of pionic degrees of freedom.

## 4. CONCLUSION

It remains extremely crucial to find a proper theoretical baseline to study the properties of strongly interacting matter and delve in sketching an outline for existence of the critical end-point. The Polyakov-Nambu-Jona—Lasinio model though provides such a platform was lacking on some cruciality. Now, with our modified model framework, that is by appropriately entangling the medium-dependent pionic contributions to PNJL model especially in the low temperature regime, the theory finds enough completion to be compared with the experimental and lattice results. The concurrence of the same are shown for various thermodynamic observables, in terms of pressure, specific heat, energy, entropy and trace anomaly that might act as suitable signatures for associated phase transitions. Also, the search for a proper theoretical model framework barring the complexities of non-zero chemical potential lattice calculations is supposed to find an answer. The incorporation of other hadrons especially for strange sector and investigation at finite chemical potentials thus get enough impetus and we intend to carry out the analysis to be presented as future aspects.

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## 6. REFERENCES

[1] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B **730**, 99 (2014).

[2] A. Bazavov *et. al.* (HotQCD Collaboration), Phys. Rev. D **85**, 054503 (2012); Phys. Rev. D **90**, 094503 (2014).

[3] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha and S. Upadhaya, Phys. Rev. D **95**, 054005 (2017).

[4] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Kreig, C. Ratti, and K. K. Szabo, J. High Energy Phys. **09**, 073 (2010).

[5] E. Megias, E. R. Arriola, and L. L. Salcedo, Phys. Rev. Lett. **109**, 151601 (2012).

[6] A. Miyahara, Y. Torigoe, H. Kouno, and M. Yahiro, Phys. Rev. D **94**, 016003 (2016).

[7] A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha, S. Samanta, and S. Upadhaya, Phys. Rev. C **99**, 045207 (2019).
[8] K. Fukushima, Phys. Lett. B **591**, 277 (2004).
[9] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73**, 014019 (2006).
[10] P. N. Meisinger and M. C. Ogilvie, Phys. Lett. B **379**, 163 (1996); Nucl. Phys. B, Proc. Suppl. **47**, 519 (1996).
[11] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **73**, 114007 (2006).
[12] S. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **75**, 094015 (2007).
[13] S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **77**, 094024 (2008).
[14] M. Ciminale, R. Gatto, N. D. Ippolito, G. Nardulli, and M. Ruggieri, Phys. Rev. D **77**, 054023 (2008).
[15] A. Bhattacharyya, P. Deb, S. K. Ghosh, and R. Ray, Phys. Rev. D **82**, 014021 (2010).
[16] G. Y. Shao, Z. D. Tang, M. Di Toro, M. Colonna, X. Y. Gao, and N. Gao, Phys. Rev. D **94**, 014008 (2008).
[17] G. Y. Shao, Z. D. Tang, M. Di Toro, M. Colonna, X. Y. Gao, N. Gao, and Y. L. Zhao, Phys. Rev. D **92**, 114027 (2015).
[18] C. A. Islam, S. Majumder, N. Haque, and M. G. Mustafa, J. High Energy Phys. **02**, 011 (2015).
[19] S. Ghosh, T. C. Peixoto, V. Roy. F. E. Serna, and G. Krein, Phys. Rev. D **93**, 045205 (2016).
[20] J. Moreira, B. Hiller, A. A. Osipov, and A. H. Blin, Int. J. Mod. Phys. A **27**, 1250060 (2012).
[21] C. Ratti, S. Robner, and W. Weise, Phys. Lett. B **649**, 57 (2007).
[22] K. Fukushima, Phys. Rev. D **77**, 114028 (2008); **78**, 039902 (E) (2008).
[23] G. A. Contrera, A. G. Grunfeld, and D. B. Blaschke, Phys. Part. Nucl. Lett. **11**, 342 (2014).
[24] X. Xin, S. Qin, and Y. Liu, Phys. Rev. D **89**, 094012 (2014).
[25] J. Hufner, S. P. Klevansky, and P. Zhuang, Ann. Phys. (N.Y.) **234**, 225 (1994).
[26] P. Zhuang, J. Hufner, and S. P. Klevansky, Nucl. Phys. (N.Y.) A **576**, 525 (1994).
[27] A. Wergieluk, D. Blaschke, Y. L. Kalinovsky, and A. V. Friesen, Phys. Part. Nucl. Lett. **10**, 660 (2013).

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