A Comparison of Fuzzy Membership Numbers on the Convergence of Numerical Approaches to Solving Integro-Differential Equations

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Abstract: In this article examines the convergences stability of Variational Iteration Method (VIM) for solving Fuzzy Volterra Integro-Differential Equations (FVIDE) of second kind under the Seikkala derivative [13]. The advantage of the proposed method in this study compared with Adomian decomposition Method (ADM). The undefined variables are represented by membership values in trapezoids and triangles. The results of the two methods are compared to demonstrate the efficacy of fuzzy numbers in terms of increasing membership values.

Keywords: Fuzzy numbers, Iterative technique, Convergence analysis

1. INTRODUCTION

The significance of differential equations with delay inequalities has grown in numerous mathematical modelling systems, including those used in biology, engineering, etc. The fuzzy concept in delay differential equations has helped eliminate the fuzziness and guesswork associated with these equations. Furthermore, many researchers have worked on these equations intending to advance the fuzzy delay differential equations field. In this regard, we note the work of Abbasbandy et al. [1], who, employing the Taylor technique, discovered the numerical solution to fuzzy delay differential equations; Vasile Lupulescu et al. [17], who, utilizing the Liu process, demonstrated the existence and uniqueness of the solution to undefined delay differential equations. Normah Maan et al. In the context of predator-prey interactions, etc., [12] analysed the stability of the steady-state solution to the fuzzy delay differential equation. To solve the linear vague delay differential equations with He's polynomials, this chapter introduces the powerful variational iteration method (VIM) tool. For various problems in both linear and non-linear classes, "He" presented the variational iteration method [5-10]. Since its iterations are direct and straightforward, it reduces the number of iterations calculated compared to other analytical methods used to solve these equations while still providing strong and simply capable solutions without sacrificing generality. A general Lagrange multiplier is used to build a correction function by combining the Laplace transform and variational theory. The advantages of the He's polynomials-based variational iteration method include its precision, ease of use, and suitability to problems arising in the physical realm. No rounding off or Adomian polynomial calculations are required.

In this work, the fuzzy Integro-differential equations are iterated using the VIM due to its many benefits. Integro-Differential equations have the generic form

$$u'(x) = f(x, u(x)), t \in [x_0, X] \quad (1)$$

with the initial condition $u(x_0) = u_0$, where u is a fuzzy function of x and $f(x)$ are given real valued functions, then $f(u(x))$ is a nonlinear function of $u(x)$. The solutions to Eq.(1) are also crisp if and only if $f(x)$ is a crisp function. However, if $f(x)$ is a fuzzy function, then this equation might only have ill-defined answers. We believe this strategy to be effective, and the numerical answer is shown to back up our claims. In this, we decompose the Fuzzy Volterra Integro-Differential Equations (FVIDE) with Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) under triangular and trapezoidal fuzzy numbers. We also discussed the convergence of variational iteration methods by analysing the numerical results.

2. PRELIMINARIES

2.1. Variation of a Function:

A variable quality $\boldsymbol{v}$ is a functional dependent on $\boldsymbol{u}(\boldsymbol{x})$ is a function to all the corresponds values in $\boldsymbol{v}$. The $\boldsymbol{v}[\boldsymbol{u}(\boldsymbol{x})]$ is defined by

$$\mathbf{P\,v}[\mathbf{u}(\mathbf{x})] = \left[\frac{\boldsymbol{\partial}}{\boldsymbol{\partial\alpha}}\,\mathbf{v}[\mathbf{u}(\mathbf{x}) + \boldsymbol{\alpha}\mathbf{Pu}]\right]_{\boldsymbol{\alpha}=\mathbf{0}}$$

2.2. Triangular Fuzzy Number:

Fuzzy set "$\bar{A}$" is the triangular fuzzy number with peak (or center) "a", left width $\alpha > 0$ and right $\beta > 0$, has the following form

$$\mu(x) = \begin{cases} 1 - \dfrac{a-x}{\alpha} & if \quad a-\alpha < x < a, \\[2ex] 1 - \dfrac{x-a}{\beta} & if \quad a < x < a+\beta, \\[2ex] 0 & otherwise. \end{cases}$$

2.3. The Trapezoidal Fuzzy Number:

The trapezoidal fuzzy number is defined by four real numbers "a, b, c, d" . A trapezoidal fuzzy number will be denoted by the membership function is defined as the

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a} & if \quad a \le x \le b, \\ 1 & if \quad b \le x \le c, \\ \dfrac{x-a}{b-a} & if \quad c \le x \le d, \\ 0 & otherwise \end{cases}$$

3. HE'S VARIATIONAL ITERATION METHOD

Now, to illustrate the basic idea of He's variable iterative method, we consider the following generalized differential equation given in the

$$Lu(t) + Nu(t) = g(t)$$

with L is a linear operator, N is a nonlinear operator, and $g(t)$ is a known analytical function. We constructing a revised functional according to the VIM as

$$u_{(n+1)}(t) = u_n(t) + \int_0^x \lambda(\eta)(Lu_n(\eta) + N\breve{u}_n(\eta) - g(\eta))d\eta,$$

Where λ is a general Lagrange multiplier, which can be identified optimally via variational theory. Now, we apply He's polynomials

$$\sum_{n=0}^{\infty} P^{(n)}u_n = u_0(x) + P\int_0^x \lambda(\eta)\left(\sum_{n=0}^{\infty} P^{(n)}\, Lu_n(\eta) + \sum_{n=0}^{\infty} P^{(n)}\, N\breve{u}_n(\eta)\right)d\eta - \int_0^x \lambda(\eta)g(\eta)\, d\eta\,.$$

The comparison of like power of P give solutions of various orders in the equation (1.3.3) will gives the approximate values. VIMHP is formulated by the coupling of VIM and He's polynomials.

4. VOLTERRA FUZZY INTEGRO-DIFFERENTIAL EQUATIONS

The Volterra integro-differential equation of the second kind is stated by,

$$\psi'(u) = f(u) + \int_a^u k(u,z),\psi(z-\tau))dz, \qquad (4.1)$$

with initial condition $\psi(a) = c_1$, where a is a constant, $k(u,z)$ is an arbitrary kernel function. The functions $F(\psi(z))$ is nonlinear function of $\psi(z)$. If $f(u)$ is a crisp function then the solutions of equation (3.1) are also crisp. However, if $f(u)$ is a fuzzy function this equation may posses only fuzzy solutions. Let us introduce α-level set of ψ, f and F to the above equation and we have,

$$\begin{aligned}
f(u,\alpha) \quad &= [\underline{f}(u,\alpha), \overline{f}(u,\alpha)],\\
\overline{\psi}(z-\tau,\alpha)) &= [G(\underline{\psi}(z,\alpha), \overline{\psi}(z,\alpha), \underline{\psi}(z-\tau,\alpha), \overline{\psi}(z-\tau,\alpha)),\\
&\quad H(\underline{\psi}(z,\alpha), \overline{\psi}(z,\alpha), \underline{\psi}(z-\tau,\alpha), \overline{\psi}(z-\tau,\alpha))],\\
\psi(u,\alpha) \quad &= [\underline{\psi}(u,\alpha), \overline{\psi}(u,\alpha)],\\
\psi(u,\alpha) \quad &= [\underline{\psi}'(u,\alpha), \overline{\psi}'(u,\alpha)], 0 \le \alpha \le 1 and\, u \in [a,b]
\end{aligned}$$

where all the derivatives are with respect to u, are fuzzy functions. Therefore, the related Volterra fuzzy integro-differential equation of (4.1) can be written as follows

$$\begin{cases}
\underline{\psi}'(u) = \underline{f}(u) + \int_a^r k(u,z),\psi(z-\tau))dz,\\
\overline{\psi}'(u) = \overline{f}(u) + \int_a^r k(u,z),\psi(z-\tau))dz,
\end{cases} \qquad (4.2)$$

for $0 \le \alpha \le 1$. Suppose $k(u,z)$ be continuous in $a \le z \le b$ and for fix u, the sign of $k(u,z)$ does not change over $[a,u]$, therefore we have

$$\underline{\psi}'(u) = \underline{f}(u) + \int_a^u k(u,z) G(\underline{\psi}(z,\alpha), \overline{\psi}(z,\alpha), \underline{\psi}(z-\tau,\alpha), \overline{\psi}(z-\tau,\alpha))dz,$$

$$\overline{\psi}'(u) = \overline{f}(u) + \int_a^r k(u,z) H(\underline{\psi}(z,\alpha), \overline{\psi}(z,\alpha), \underline{\psi}(z-\tau,\alpha), \overline{\psi}(z-\tau,\alpha))dz, \tag{4.3}$$

with subject to initial condition $[\underline{\psi}(z,\alpha), \overline{\psi}(z,\alpha)] = [\underline{b}_1(\alpha), \overline{b}_1(\alpha)]$ for each $0 \le \alpha \le 1$ and $z \in [a,b]$. We can see that equation (4.3) are system of Volterra fuzzy integro-differential equations in the crisp case for each $0 \le \alpha \le 1$.

5. UNIQUENESS AND CONVERGENCE OF HE'S VARIATIONAL ITERATION METHOD

Theorem 5.1. If a functional $v[u(x)]$; has a variation, achieves a maximum or a minimum at $u = u_o(x)$, where $u(x)$ is an interior point of the domain of definition of the functional, then at $u = u_o(x)$ [5],

$Pv = 0.$

5.2. Uniqueness theorem

Theorem 5.2.1. Let f satisfy

$$|f(x,v) - f(x,\overline{v})| \le g(x,|v - \bar{v}|), x \ge 0, v, \bar{v} \in R,$$

Proof: The proof of this theorem is similar to Theorem [11, 18].

Theorem 5.2.2. The problem (4.1) has a unique solution, whenever $0 < \aleph < 1$, where $\aleph = (m_2 + m)T$ and the constants m_2 and m are arbitrary constants, i.e., it is possible to find numbers $m_1, m_2 > 0$ such that $||L_u|| \le m_1||u||, ||R_u|| \le m_2||u||$. The nonlinear term $g(u)$ is Lipschitz continuous with $|g(u) - g(v)| \le m|u - v|, \forall t \in J = [0,T]$, for any arbitrary constants $m > 0$.

Proof: Since, the solution of Eq. (4.1) can be written in the following form

$$u = f(t) - L^{-1}[R(u) + g(u)],$$

where f(t) is the solution of the homogeneous equation Lu = 0, the inverse operator L^{-1} is defined by $L^{-1}(.) = \int_0^x (.)dt$.

Now let, u and u* be two different solutions to Eq.(4.1) then by using the above equation, we get

$$|u - u^*| = \left|\int_0^x [R(u-u^*) + g(u) - g(u^*)]dt\right|$$

$$\le \int_0^x [|R(u-u^*)| + |g(u) - g(u^*)|]dt$$

$$\le (m_2|u-u^*| + m|u-u^*|)T$$

$$\le \alpha|u-u^*|$$

From which we get $(1-\alpha)|u-u^*| \le 0$. Since $0 < \aleph < 1$, then $|u-u^*| = 0$ impies, $u - u^*$.

5.3. Convergence theorem

Theorem 5.3.1. (Banach's fixed point theorem) Assume that X be a Banach space and $A : X \to X$

is a nonlinear mapping and suppose that,

$$||\underline{A}[u] - \underline{A}[u^*]|| \le \gamma||\underline{u} - \underline{u^*}||; ||\overline{A}[u] - \overline{A}[u^*]|| \le \gamma||\overline{u} - \overline{u^*}||, \forall u, u^* \in X,$$

For some constant $\gamma < 1$. Then A has a unique fixed point. Furthermore, the sequence

$$u_{n+1} = A[u_n];\ u_{\ n+1} = A[u_n]$$

with an arbitrary choice of $\underline{u_0}$; $\overline{u_0} \in X$, converges to the fixed point A and

$||\underline{u_k} - \underline{u_l}|| \le ||\underline{u_1} - \underline{u_0}||\frac{\gamma^l}{1-\gamma}$; $||\overline{u_k} - \overline{u_l}|| \le ||\overline{u_1} - \overline{u_0}||\ \frac{\gamma^l}{1-\gamma}$

Proof: Denoting $(C[J], ||.||)$ Banach space of all continuous functions on J with the norm defined by

$$||f(t)|| = max_{t\in J}\ |f(t)|.$$

We are going to prove that the sequence u_k is a Cauchy sequence in this Banach space

$$||\underline{u_k} - \underline{u_l}|| = max_{\ t\in J}|u_k + u_l|;$$

$$||\overline{u_k} - \overline{u_l}|| = max_{t\in J}\ |u_k + u_l|$$

$$\le\ max_{\ t\in J} \int_0^x \left[(m_1 + m_2 + m)\left(\underline{u_{k-1}} + \underline{u_{l-1}}\right)\right] d\eta;$$

$$\le\ max_{\ t\in J} \int_0^x [(m_1 + m_2 + m)(\overline{u_{k-1}} + \overline{u_{l-1}})]\ d\eta;$$

$$\le \gamma||\underline{u_{k-1}} - \underline{u_{l-1}}||; \gamma||\overline{u_{k-1}} - \overline{u_{l-1}}||$$

$k = l + 1\ then$

$$||\underline{u_{l+1}} - \underline{u_l}|| \le \gamma||\underline{u_l} - \underline{u_{l+1}}|| \le \gamma^2||\underline{u_{l+1}} - \underline{u_{l+1}}|| \le \ldots \le \gamma^l||\underline{u_1} - \underline{u_0}||;$$

$$||\overline{u_{l+1}} - \overline{u_l}|| \le \gamma||\overline{u_l} - \overline{u_{l+1}}|| \le \gamma^2||\overline{u_{l+1}} - \overline{u_{l+1}}|| \le \ldots \le \gamma^l||\overline{u_1} - \overline{u_0}||$$

From the triangular inequality we have

$$||\underline{u_k} - \underline{u_l}|| \le \gamma||\underline{u_{l+1}} - \underline{u_l}|| \le \gamma^2||\underline{u_{l+1}} - \underline{u_l}|| \le \ldots \le \gamma^l||\underline{u_k} - \underline{u_{k-1}}||;$$

$$||\overline{u_k} - \overline{u_l}|| \le \gamma||\overline{u_{l+1}} - \overline{u_l}|| \le \gamma^2||\underline{u_{l+1}} - \underline{u_l}|| \le \ldots \le \gamma^l||\overline{u_k} - \overline{u_{k-1}}||$$

$$\le \left\|\underline{u_1} - \underline{u_0}\right\| \gamma^q \frac{1 - \gamma^{k-l-1}}{1 - \gamma}; ||\overline{u_1} - \overline{u_0}||\ \gamma^q \frac{1 - \gamma^{k-l-1}}{1 - \gamma}$$

Since $0 < \gamma < 1$ so, $(1 - \gamma^{k-l}) < 1$ then

$$\left\|\underline{u_k} - \underline{u_l}\right\| \le \left\|\underline{u_1} - \underline{u_0}\right\| \frac{\gamma^l}{1 - \gamma}; ||\overline{u_k} - \overline{u_l}|| \le ||\overline{u_1} - \overline{u_0}||\frac{\gamma^l}{1 - \gamma};$$

But $\left\|\underline{u_k} - \underline{u_l}\right\| \le \infty$; $\left|\,|\overline{u_k} - \overline{u_l}|\right| \le \infty$ so, as $l \to \infty$ then $\left\|\underline{u_k} - \underline{u_l}\right\| \longrightarrow 0$; $\left|\,|\overline{u_k} - \overline{u_l}|\right| \longrightarrow 0$. We conclude that $\underline{u_l}$; $\overline{u_l}$ is a Cauchy sequence in $C[J]$ so, the sequence convergent. Hence the proof.

6. MATLAB ALGORITHM

Here's a MATLAB code template that demonstrates the implementation of the variational iteration method for solving a fuzzy nonlinear DDE:

Step 1: Represent the Fuzzy Differential Equation

$$D(u(t)) = f(t, u(t)), \qquad u(t_0) = u_0$$

Step 2: Apply the Variational Iteration Method

$$u(t) = sum\_\{n = 0\}\hat{}\{N\}\, u\_n(t)$$

Step 3: Implement the Algorithm in MATLAB

```
function u = fuzzy_diff_eq_variational_iteration(f, tspan, u0, N)
    % Inputs:
    % f: Function handle representing the fuzzy function f(t, u(t))
    % tspan: Time span [t0, t_end] for solving the fuzzy differential equation
    % u0: Initial condition u(t0)
    % N: Number of iterations for the variational iteration method

    % Step 1: Set up time span and initial condition
    t0 = tspan(1);
    t_end = tspan(2);
    t = linspace(t0, t_end, 100); % Adjust the number of time steps as needed

    % Step 2: Initialize u(t) and perform variational iteration method
    u = zeros(1, length(t));
    u(1) = u0;

    for n = 1:N
        % Construct the linear operator L
        L = @(u_n) diff(u_n, 1) - f(t, u_n);

        % Define the initial guess for the nth term (can be improved)
        u_n = u(n);

        % Use MATLAB's built-in solver to find the nth approximation
        % Adjust options as needed (e.g., 'RelTol', 'AbsTol', etc.)
        [t, u_n] = ode45(L, tspan, u_n);

        % Update the solution using the nth approximation
        u = u + u_n';
    end
end
```

7. NUMERICAL EXAMPLE

Here we demonstrate the effectiveness of He's method of repeated variables in solving differential equations with ambiguous initial conditions, finding approximate solutions and generating tables with graphs for analysis. Here we use triangular and trapezoidal fuzzy numbers for our analysis.

Example 7.1. Let us consider the fuzzy linear Volterra Integro-differential equation of the following form

$$u''(x) = x + \int_0^x (x-t)(t)dt, u(0) = 0, u'(0) = 1$$

7.1.1 Solving Using the Triangular Fuzzy Number

$u(0) = (\alpha - 1, 1 - \alpha);\ u'(0) = (\alpha, 2 - \alpha)$

Exact solution

$$\underline{u}(x) = \frac{\alpha - 1}{2}[sinx + cosx + coshx] + \frac{\alpha + 1}{2} sinhx;$$

$$\overline{u}(x) = \frac{1 - \alpha}{2}[sinx + cosx + coshx] + \frac{3 - \alpha}{2} sinhx$$

Using the VIM method,

Left hand

$$\begin{aligned} u_{n+1}(x) = {} & -1 + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^7}{5040} - \frac{x^8}{40320} + \frac{x^{11}}{39916800} - \frac{x^{12}}{479001600} + \frac{x^{15}}{1307674368000} - \frac{x^{16}}{20922789888000} \\ & + \frac{x^{19}}{121645100408832000} - \frac{x^{20}}{2432902008176640000} \\ & + a\left(1 + x + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^8}{40320} + \frac{x^9}{362880} + \frac{x^{12}}{479001600} + \frac{x^{13}}{6227020800} + \frac{x^{16}}{20922789888000} \right. \\ & \left. + \frac{x^{17}}{355687428096000} + \frac{x^{20}}{2432902008176640000} + \frac{x^{21}}{51090942171709440000}\right) + \cdots \end{aligned}$$

Right hand

$$\begin{aligned} u_{n+1}(x) = {} & 1 - a + (2 - a)x + \frac{x^7}{5040} + \frac{x^8}{40320} - \frac{ax^8}{40320} + \frac{x^9}{181440} - \frac{ax^9}{362880} + \frac{x^{11}}{39916800} + \frac{x^{12}}{479001600} - \frac{ax^{12}}{479001600} \\ & + \frac{x^{13}}{3113510400} - \frac{ax^{13}}{6227020800} + \frac{x^{15}}{1307674368000} + \frac{x^{16}}{20922789888000} - \frac{ax^{16}}{20922789888000} \\ & + \frac{x^{17}}{177843714048000} - \frac{ax^{17}}{355687428096000} + \frac{x^{19}}{121645100408832000} + \frac{x^{20}}{2432902008176640000} \\ & - \frac{ax^{20}}{2432902008176640000} + \frac{x^{21}}{25545471085854720000} - \frac{ax^{21}}{51090942171709440000} \\ & - \frac{1}{120}x^3(-20 + 5(-1 + a)x + (-2 + a)x^2) + \cdots \end{aligned}$$

Table 1 Numerical results of problem 1 at t = 1 (Triangular Fuzzy Number)

Exact		**ADM**		**VIM**	
Left	**Right**	**Left**	**Right**	**Left**	**Right**
-0.8748	3.2252	-0.8748263659	3.2252287530	-0.8748263659	3.2252287530
-0.4648	2.8152	-0.4648208540	2.8152232410	-0.4648210000	2.8152200000
-0.0548	2.4052	-0.0548153421	2.4052177290	-0.0548153000	2.4052200000
0.3552	1.9952	0.3551901698	1.9952122170	0.3551900000	1.9952100000
0.7652	1.5852	0.7651956817	1.5852067600	0.7651960000	1.5852100000
1.1752	1.1752	1.1752011940	1.1752011940	1.1752011940	1.1752011940

VIM Error	
Left	**Right**
2.6365900E-05	-2.87530000E-05
2.1000000E-05	-2.00000000E-05
1.5300000E-05	-2.00000000E-05
1.0000000E-05	-1.00000000E-05
4.0000000E-06	-1.00000000E-05
-1.1940000E-06	-1.19400000E-06

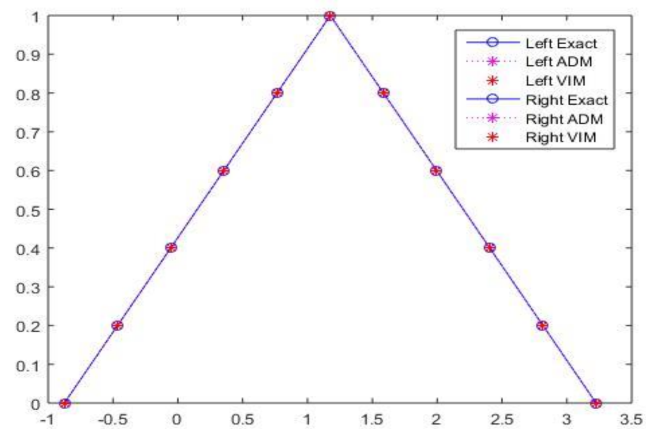

Figure 1

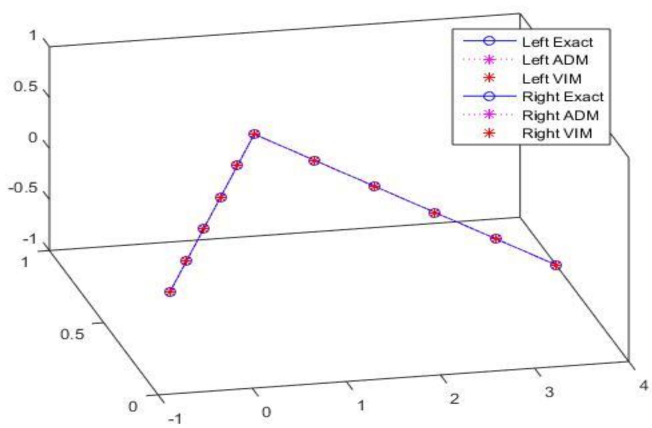

Figure 2

Figure 1 & Figure 2 Approximate solution and exact solution (line) at t = 1.0 for 7.1.1

7.1.2 Solving Using the Trapezoidal Fuzzy Number

$u(0) = (0.0125 + 0.01\alpha, 0.1 - 0.1\alpha)$; $u'(0) = (0.8 + 0.125\alpha, 1.1 - 0.1\alpha)$

Exact solution

$\underline{u}(x) = (0.00625 + 0.005\alpha)[sinx + cosx + coshx] + (0.9 + 0.0625\alpha)sinhx$

$\overline{u}(x) = (0.05 - 0.05\alpha)[sinx + cosx + coshx] + (1.05 - 0.05\alpha)sinhx$

Using the VIM method,

Left hand

$$u_{n+1}(x) = 0.0125 + 0.8x + 0.16666666666666666x^3 + 0.0005208333333333333x^4 + 0.006666666666666668x^5 + 0.00019841269841269836x^7 + 3.100198412698416 \times 10^{-7}x^8 + 0.0000022045855379 18872x^9 + 2.505210838544173 \times 10^{-8}x^{11} + 2.609594623483511 \times 10^{-11}x^{12} + 1.28472350694573 \times 10^{-10}x^{13} + 7.647163731819805 \times 10^{-13}x^{15} + 5.974346665484241 \times 10^{-16}x^{16} + 2.249165803476414 \times 10^{-15}x^{17} + 8.220635246624344 \times 10^{-18}x^{19} + 5.137897029140196 \times 10^{-21}x^{20} + 1.565835285071305 \times 10^{-20}x^{21} + a(0.01 + 0.125x + 0.00041666666666666667x^4 + 0.0010416666666666669x^5 + 2.480158730158732 \times 10^{-7}x^8 + 3.444664902998239 \times 10^{-7}x^9 + 2.087675698786807 \times 10^{-11}x^{12} + 2.007380479602705 \times 10^{-11}x^{13} + 4.77947733238739 \times 10^{-16}x^{16} + 3.514321567931908 \times 10^{-16}x^{17} + 4.110317623312183 \times 10^{-21}x^{20} + 2.446617632923907 \times 10^{-21}x^{21}) + \cdots$$

Right hand

$$u_{n+1}(x) = 0.1 - 0.1a + (1.1 - 0.1a)x + \frac{x^3}{6} + 0.004166666666666667x^4 - 0.004166666666666667ax^4 + 0.009166666666666667x^5 - 0.0008333333333333335ax^5 + 0.00019841269841269836x^7 + 0.0000002480158730158732x^8 - 0.0000002480158730158732ax^8 + 0.0000003031305114638446x^9 - 2.755731922398591 \times 10^{-7}ax^9 + 2.505210838544173 \times 10^{-8}x^{11} + 2.087675698786808 \times 10^{-10}x^{12} - 2.087675698786808 \times 10^{-10}ax^{12} + 1.76649482205038 \times 10^{-10}x^{13} - 1.605904383682162 \times 10^{-11}ax^{13} + 7.647163731819805 \times 10^{-13}x^{15} + 4.779477332387393 \times 10^{-15}x^{16} - 4.779477332387393 \times 10^{-15}ax^{16} + 3.092602979780074 \times 10^{-15}x^{17} - 2.811457254345518 \times 10^{-16}ax^{17} + 8.220635246624319 \times 10^{-18}x^{19} + 4.110317623312156 \times 10^{-20}x^{20} - 4.110317623312156 \times 10^{-20}ax^{20} + 2.153023516973045 \times 10^{-20}x^{21} - 1.957294106339131 \times 10^{-21}ax^{21} + \cdots$$

Table 2Numerical results of problem 1 at t = 1(Trapezoidal Fuzzy Number)

Exact		ADM TRA		VIM TRA	
Left	**Right**	**Left**	**Right**	**Left**	**Right**
1.0759614113	1.3802039496	0.9865551192	1.3802039496	0.9865551192	1.3802039496
1.0935762802	1.3392033984	1.0138469043	1.3392033984	1.0138469043	1.3392033984
1.1111911490	1.2982028472	1.0411386895	1.2982028472	1.0411386895	1.2982028472
1.1288060179	1.2572022960	1.0684304747	1.2572022960	1.0684304747	1.2572022960
1.1464208867	1.2162017448	1.0957222599	1.2162017448	1.0957222599	1.2162017448
1.1640357555	1.1752011936	1.1230140450	1.1752011936	1.1230140450	1.1752011936

VIM Error	
Left	**Right**
0.0894062921	-2.87530000E-15
0.0797293758	-2.00000000E-15
0.0700524595	-2.00000000E-15
0.0603755432	-1.00000000E-15
0.0506986268	-1.00000000E-15
0.0410217105	-1.19400000E-15

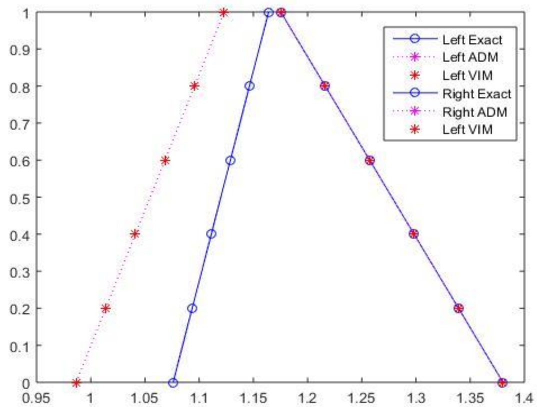

Figure 3

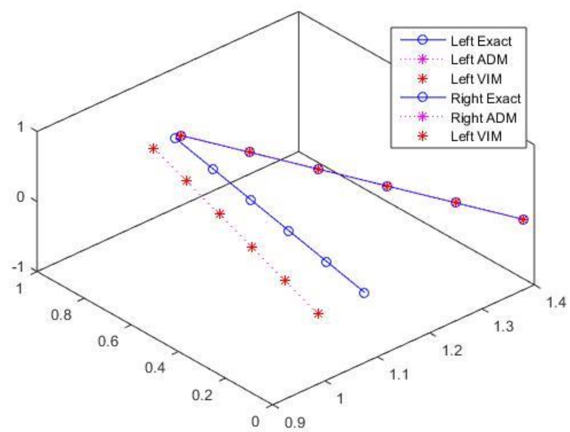

Figure 4

Figure 3 & Figure 4 Approximate solution and exact solution (line) at t = 1.0 for 7.1.2

8. CONCLUSION

Finally, this article proposes to study the VIM algorithm using the He polynomial to solve the FVID equation within the concept of the Seikkala derivative [13]. The proposed methodological solution compared with ADM to show the effective convergence of VIM. From numerical results, we have seen that the proposed strategy matches fuzzy numbers accurately, reliably, and consistently. The result obtained is processed graphically.

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