International Journal of Membrane Science and Technology, 2023, Vol. 10, No. 2, pp 2649-2657

A New Approach of Presenting Lindley Expontiated Gumbel Distribution with Application to Environmental Data

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Abstract: The Lindley exponentiated Gumbel (LEGu) Distribution, a new family of probability distribution developed for modelling environmental data is introduced in this research paper. The investigation defines the new distribution's basic statistical properties, such as its shape, density function, hazard rate function, moment generation function, and maximum likelihood estimates of its model parameters. The study shows that, in comparison to other baseline distributions and comparators, increasing the number of parameters in the distribution improves its robustness and adaptability, making it more flexible in accommodating different types of data. The model is now more adaptable and can handle a wider range of data sets, making it a useful and effective tool for environmental modelling. The creation of the Lindley exponentiated Gumbel Distribution and the statistical qualities that go along with it bring up new possibilities for the analysis of environmental data, making a significant contribution to the larger initiatives in climate research and decision-making. The results of this study have the potential to advance our comprehension of environmental mechanisms and raise the precision of climate assessments and projections.

Keywords: Environment, Exponentiated Gumbel, Gumbel, Lindley, Temperature.

1. INTRODUCTION

Over the past 20 years, countries from all over the world have worked together at the local, national, and international levels to protect the environment while pursuing sustainable development and the reduction of poverty. In order to encourage equitable growth, guarantee food security, and enhance human well-being, it is now essential to manage renewable natural resources, watersheds, landscapes, and coastal areas properly. A healthy and productive life depends on a clean environment, which frees up resources to be used for development rather than pollution abatement. The global ecosystem is essential in maintaining the quality of the air, water, and soil while acting as a protective barrier against extreme weather conditions and climate change. Adopting tactics like efficient waste management programs, eco-friendly tax policies, greener financial markets, and effective natural resource management on a global scale are essential for achieving sustainable growth.

The actions taken to combat climate change and nature should complement one another and encourage inclusive, resilient, and green growth. The advantages of wise policies can be greatly increased by using the advantages of natural carbon sequestration services. While the Lindley distribution can describe data with a monotonic and growing failure rate because it just has one scale parameter, there is a need for more adaptable substitutes for modelling. “Numerous generalizations of the Lindley distribution have been investigated by researchers, such as the three-parameter version developed and examined [15]. The general Lindley distribution presented by [11] and the weighted distribution of Lindley by [7] have also been studied, offering thorough insights into their mathematical properties. The Poisson-Lindley distribution for the Exponential Poisson was first described by [3]. The Power Lindley distribution was made possible by [6]. In 2013, [13] contributed to the expanding body of knowledge in this area by introducing a two-parameter Lindley distribution.

As a specific case, the Lindley distribution is frequently used to simulate waiting and survival data. [2] offered a new weighted Lindley distribution (WL) in an effort to broaden its applicability, and [14] suggested the generalized reverse Lindley distribution. Many research has focused on circumstances when data shows a drop-in risk rate and failure rate, resulting in reversing bathtubs (UBT) shapes. [12] for instance, noted a decline in the failure rate of airplane air conditioning systems. [8] investigated the North Anatolia fault zone earthquakes and discovered a suitable distribution with low cause failure rates. It is important to note that risk ratios for inverse probability distributions exhibit UBT forms. Different distributions have been thought of as alternatives, including uniform, power, Bates, arcsine, Kumaraswamy, Topp-Leone, beta, triangle, raised cosine, and Von Mises".

The exponentiated Gumbel distribution, developed from the widely used Gumbel distribution for modelling extreme value data, was introduced in the field of skewed distributions by [10]. This research proposes a novel family of probability distributions known as Lindley Exponentiated Gumbel Distributions (LEGu), building on the exponentiated Gumbel distribution, which has found practical applications in climate modelling, addressing issues such as global warming, flood frequency analysis, ocean modelling, rainfall modelling, and wind speed modelling.

2. LINDLEY-G FAMILY OF DISTRIBUTIONS

By extending the transformer (T-X) generator first described by [1], [4] created the Lindley-G family of distributions, a broader class of continuous probability distributions. This new family was created by integrating the cumulative density function (CDF) and probability distribution function (PDF) of the Lindley density function, and it is shown as follows:

i. Lindley - G family of Distribution

$$F(x)=1-\left[1-\frac{\theta}{\theta+1}\left[\log\left[1-G(x)\right]\right]\right]\left[1-G(x)\right]^{\theta} \tag{1}$$

and

$$f(x)=\frac{\theta^{2}}{\theta+1}g(x)\left[1-\log\left[1-G(x)\right]\right]\left[1-G(x)\right]^{\theta-1} \tag{2}$$

ii. Exponentiated Gumbel Distribution

Exponentiated Gumbel (EG) distribution, developed by [10] as a generalization of the Gumbel distribution, with cdf and pdf given as:

$$G(x)=1-\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha} \tag{3}$$

and

$$g(x)=\frac{\alpha}{\sigma}\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}-\exp\left(\frac{-x-\mu}{\sigma}\right) \tag{4}$$

iii. Lindley Exponentiated Gumbel Distribution

Equations (3) and (4) are introduced into equations (1) and (2), respectively, to produce the Lindley Exponentiated Gumbel distribution's CDF and PDF, respectively.

$$F(x)=1-\left[1-\frac{\theta}{\theta+1}\left[\log\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]\right]\left[\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]^{\theta} \tag{5}$$

and

$$f(x)=\frac{\theta^{2}\alpha}{\sigma(\theta+1)}\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\left[1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]\right]\right]$$
$$\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\exp(-x)\left[1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]\right]^{\theta-1} \tag{6}$$

$-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\alpha,\sigma,\theta > 0$.

Where α,θ are the shape parameters controlling the skewness and kurtosis, σ is the scale parameter and μ is the location parameter.

Without loss of generality, we assume $\mu=0$ and $\sigma=1$ then equations (5) and (6) become

$$F(x)=1-\left[1-\frac{\theta}{\theta+1}\left[\log\left[1-\exp\left\{-\exp\left(-x\right)\right\}\right]^{\alpha}\right]\right]\left[\left[1-\exp\left\{-\exp\left(-x\right)\right\}\right]^{\alpha}\right]^{\theta} \tag{7}$$

$$f(x)=\frac{\theta^{2}\alpha}{(\theta+1)}\left[1-\exp\left\{-\exp\left(-x\right)\right\}\right]^{\alpha-1}\left[1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp\left(-x\right)\right\}\right]^{\alpha}\right]\right]\right]$$
$$\exp\left\{-\exp\left(-x\right)\right\}\exp(-x)\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]^{\theta-1} \tag{8}$$

3. TEST OF VALIDITY: THE SHAPE

Plot of the pdf and cdf of LEGu Distribution

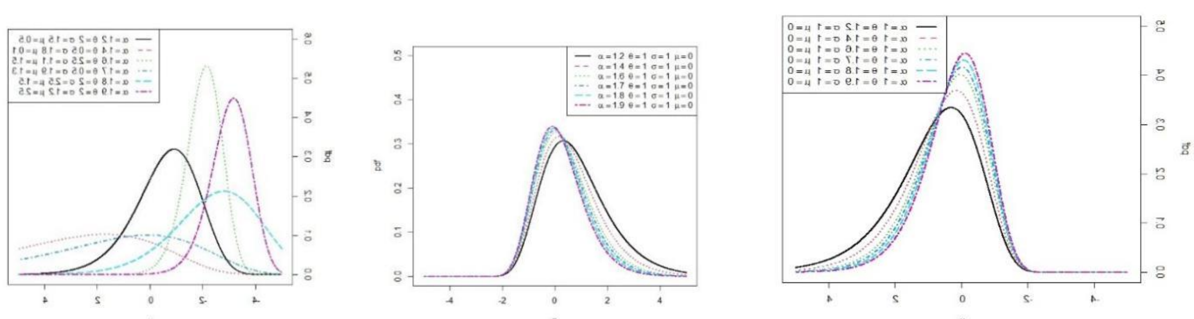

Figure 1

The probability density function (PDF) graphs are shown in Figure 1 for various parameter values. The graphs have approximately symmetrical patterns and varies in kurtosis and skewness.

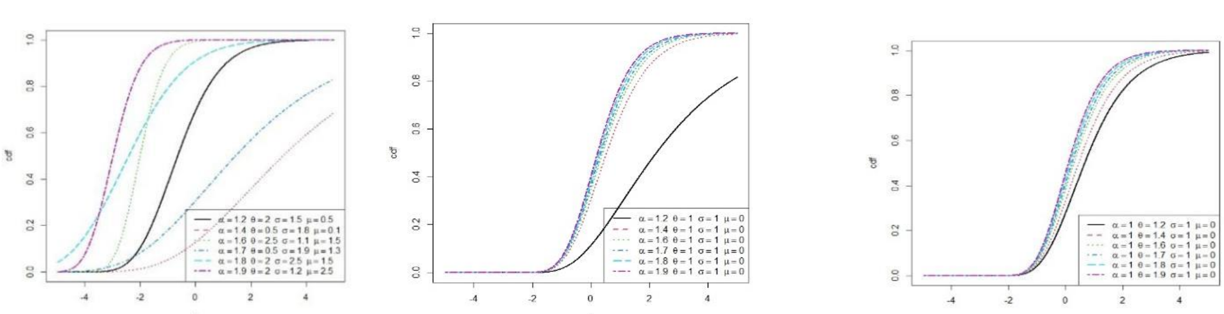

Figure 2

The cumulative distribution function (CDF) graphs for various parameter values are shown in Figure 2. The LEGu distribution is a legitimate density function because the plots have a S shape that goes from zero to one.

4. PROPERTIES OF LINDLEY EXPONENTIATED GUMBEL (LEGu) DISTRIBUTION

Some properties of the newly designed distribution, LEGu, are provided in this section.

i. Survival function

The likelihood of an item not failing before a certain time is known as the survival function. It is characterized by

$$s(x) = P(X > x) = 1 - F(x) \tag{9}$$

Where:

S(x) is the survival function at time x

P(X > x) is the probability that the random variable X (representing the time to event or failure) is greater than x.

The survival function of LEGu distribution is given as

$$s(x) = \left[1 - \frac{\theta}{\theta+1}\left[\log\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]\left[\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]^{\theta} \tag{10}$$

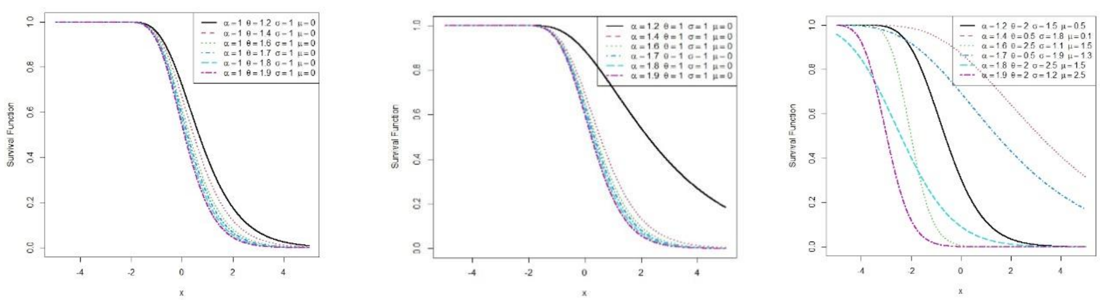

Figure 3: Survival function plots for various parameter values.

The survival function has a reverse S shape, showing that the probability of survival drops at first before levelling off over time. The plot begins at one, reflecting the entire likelihood of surviving at time zero, and ends at zero, representing the ultimate decline in survival probability to zero through time. This aspect of the survival function, which captures the shifting pattern of survival probability for the specified parameters, distinguishes the LEGu distribution.

ii. Hazard function

The Hazard rate function (Hrf) is given as

$$h(x) = \frac{f(x)}{1-F(x)} = \frac{f(x)}{s(x)} \tag{11}$$

The hazard function of LEGu distribution is given as

$$h(x) = \frac{\begin{array}{r}\frac{\theta^4\alpha}{\theta+1}\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha-1}\exp\left\{-\exp(-x)\right\}\exp(-x)\left[1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]\right] \\ \times\left[1-\left[1-\left[1-\exp\left\{-\exp(x)\right\}\right]^{\alpha}\right]\right]^{\theta-1}\times\left[1-\log\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\end{array}}{\left[\theta+1-\theta\left[\log\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]\left[\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]} \tag{12}$$

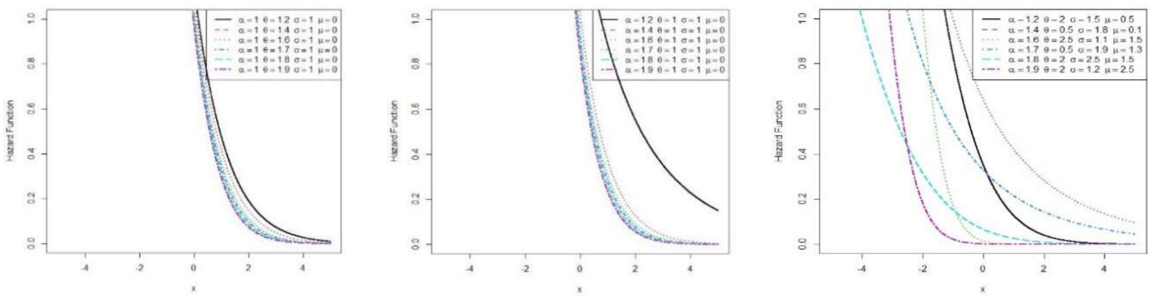

Figure 4 shows plots of the LEGu distribution's hazard function.

The hazard function has a L shape, which denotes that the hazard rate function changes with time. The plot demonstrates that as time goes on, the hazard rate initially drops and eventually stays constant. This characteristic of the hazard function is a key feature of the LEGu distribution, capturing the varying risk of failure over time for the given parameters.

iii. Moment of the newly developed distribution (LEGu)

$$E(X^r) = \int_0^\infty x^r f(x)dx \tag{13}$$

$$E(X^r) = \sum_{i,j,k,d,a,p=0}^{\infty} \psi_v \int_0^\infty x^r \exp(-x)^{p+1}dx \tag{14}$$

Where $\int_0^\infty x^r \exp(-x)^{p+1}dx = \Gamma(r+p+1)$ (15)

Therefore

$$E(X^r) = \sum_{i,j,k,d,a,p=0}^{\infty} \psi_v \Gamma(r+p+1) \tag{16}$$

The mean of the distribution is obtained by setting r=1 in equation (16)

$$E(X) = \sum_{i,j,k,d,a,p=0}^{\infty} \psi_v \Gamma(p+2) \tag{17}$$

iv. Moment Generating Function (Mgf)

$$M_x(t) = \int_0^\infty e^{tx} f(x)dx \tag{18}$$

Consider the expansion

$$e^{tx} = \sum_{m=0}^{\infty} \frac{(tx)^m}{m!}, \tag{19}$$

then the mgf of the LEGu distribution follows from the moment as

$$M_x(t) = \frac{\sum_{i,j,k,d,a,p=0}^{\infty} \sum_{m=0}^{\infty} t^m \psi_v \Gamma(m+p+1)}{m!} \tag{20}$$

5. METHOD OF ESTIMATION

Maximum Likelihood Estimation

"This section presents the maximum likelihood estimators known to give maximum information about the population parameters, therefore this section presents the maximum likelihood estimates (MLEs) of the parameters that are inherent within the distribution function"

Let $X_1, X_2, \ldots X_n$ be random variables of the LEGu distribution of size n. Then sample log-likelihood function of the LEGu distribution is obtained as

$$\log L = 2n\log\theta - n\log(\theta+1) + (\alpha-1)\sum_{i=1}^{n}\log\left[1-\exp\left\{-\exp(-x)\right\}\right] - \sum_{i=1}^{n}\exp(-x) - \sum_{i=1}^{n}x_i$$
$$+\sum_{i=1}^{n}\log\left[1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]\right] + \theta - 1\sum_{i=1}^{n}\log\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right] \quad (21)$$

Differentiating equation (21) with respect to each parameter and equating to zero, we have

$$\frac{d\log L}{d\theta} = \frac{2n}{\theta} - \frac{n}{\theta+1} + \sum_{i=1}^{n}\log\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right] \quad (22)$$

$$\frac{d\log L}{\alpha} = \sum_{i=1}^{n}\log\left[1-\exp\left\{-\exp(-x)\right\}\right] - \sum_{i=1}^{n}\frac{\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\log\left[1-\exp\left\{-\exp(-x)\right\}\right]}{1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]}$$
$$-\theta-1\sum_{i=1}^{n}\frac{\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\log\left[1-\exp\left\{-\exp(-x)\right\}\right]}{1-\left[1-\left[1-\left[1-\exp\left\{-\exp(-x)\right\}\right]^{\alpha}\right]\right]} \quad (23)$$

$$\frac{d\log L}{d\mu} = (\alpha+1)\sum_{i=1}^{n}\frac{\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}\frac{1}{\sigma}\exp\left(\frac{-x_i}{\sigma}\right)\exp\left(\frac{\mu}{\sigma}\right)}{1-\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}} - \sum_{i=1}^{n}\frac{1}{\sigma}\exp\left(\frac{-x_i}{\sigma}\right)\exp\left(\frac{\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(\frac{x_i-1}{\sigma}\right)$$
$$+\sum_{i=1}^{n}\frac{\alpha\left[1-\exp\left\{-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}\frac{1}{\sigma}\exp\left(\frac{-x_i}{\sigma}\right)\exp\left(\frac{\mu}{\sigma}\right)}{1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]\right]}$$
$$+(\theta-1)\sum_{i=1}^{n}\frac{\alpha\left[1-\exp\left\{-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}\frac{1}{\sigma}\exp\left(\frac{-x_i}{\sigma}\right)\exp\left(\frac{\mu}{\sigma}\right)}{1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]} \quad (24)$$

$$\frac{d\log L}{\sigma}=\frac{-n}{\sigma}+(\alpha-1)\sum_{i=1}^{n}\frac{\exp\left[-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right]\exp\left(\frac{-x_i-\mu}{\sigma}\right)\frac{x_i-\mu}{\sigma^2}}{1-\exp\left[-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right]}-\sum_{i=1}^{n}\exp\left(\frac{-x_i-\mu}{\sigma}\right)\frac{x_i-\mu}{\sigma^2}-\sum_{i=1}^{n}\frac{x_i-\mu}{\sigma^2}$$

$$+\sum_{i=1}^{n}\frac{\alpha\left[1-\exp\left\{-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}\exp\left(\frac{-x_i-\mu}{\sigma}\right)\frac{x_i-\mu}{\sigma^2}}{1-\log\left[1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]\right]}$$

$$+(\theta-1)\sum_{i=1}^{n}\frac{\alpha\left[1-\exp\left\{-\exp\left(\frac{-x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp(\frac{-x_i-\mu}{\sigma})\right\}\exp\left(\frac{-x_i-\mu}{\sigma}\right)\frac{x_i-\mu}{\sigma^2}}{1-\left[1-\left[1-\exp\left\{-\exp\left(\frac{x_i-\mu}{\sigma}\right)\right\}\right]^{\alpha}\right]} \quad (25)$$

Analytical approaches have difficulty obtaining the precise solution of the Maximum Likelihood Estimator (MLE) for the unknown parameters due to the complexity of equations (22), (23), (24), and (25). Therefore, a useful strategy is to use the non-linear Newton-Raphson algorithm, which offers an exact numerical solution to maximize the above-mentioned probability function.

6. APPLICATION

In this section, environmental data were used to illustrate the use of the four parameter Lindley Exponentiated Gumbel distribution.

82 observations on the lowest July temperatures in Uppsala, Sweden, from 1900 to 1981 make up the data. The following information was taken from [9]:

5.5, 6.7, 4.0, 7.9, 6.3, 9.0, 6.2, 7.2, 2.1, 4.9, 6.6, 6.3, 6.5, 8.7, 10.2, 10.8, 9.7, 7.7, 4.4, 9.0, 8.4,9.7,6.9,6.7,8.0,10.0,11.0,7.9,12.9,5.5,8.3,9.9,10.4,8.7,9.3,6.5,8.3,11.0,11.3,9.2,11.0,7.7,9.2,6.6,7.1,8.2,10.4, 10.8,10.2,9.8,7.3,8.0,6.4,9.7, 11.0,10.7, 9.4, 8.1, 8.2, 7.4, 9.0, 9.9, 9.0, 8.6, 7.0, 6.9, 11.8, 8.2, 7.0, 9.7, 8.2, 7.6, 10.5, 11.3, 7.4, 5.7, 8.6, 8.8, 7.9, 8.1, 9.0, 12.1.

Estimating equation (8) above, using the data set above, we have our result displayed in table 1 below.

THE COMPARATORS

The pdf of the comparators considered are:

Exponentiated Gumbel (EGu) distribution

$$f(x)=\frac{\alpha}{\sigma}\left[1-\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}\right]^{\alpha-1}\exp\left\{-\exp\left(\frac{-x-\mu}{\sigma}\right)\right\}-\exp\left(\frac{-x-\mu}{\sigma}\right) \quad (26)$$

Exponentiated Generalized Gumbel (EGGu) distribution [5]

$$f(x)=\frac{\alpha\theta}{\sigma}\exp\left\{-\left[\frac{x-\mu}{\sigma}+\exp\left(-\frac{x-\mu}{\sigma}\right)\right]\right\}\left\{1-\exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]\right\}^{\alpha-1}\left\{1-\left\{1-\exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right]\right\}^{\alpha}\right\}^{\theta-1} \quad (27)$$

Gumbel (Gu) distribution

$$f(x)=\frac{1}{\sigma}\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\exp\left(-\frac{x-\mu}{\sigma}\right) \quad (28)$$

Table 1: The MLEs and Information Criteria of the models based on data Set above

Models	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\mu}$	$\hat{\sigma}$	$-ll$	AIC	BIC
LEGu	5.3068	14.7339	-20.7243	7.9292	170.5640	349.1281	358.7549
EGu	87.0381	-	-21.3815	8.1644	171.8608	349.7216	358.9417
EGGu	0.2737	7.5981	-18.1048	4.4548	171.8472	351.6945	361.3213
Gu	-	-	-7.3998	2.1472	182.9826	369.9653	374.7787

In comparison to the other models, the LEGu distribution offers the best fit to the data. Table 1 shows that when compared to the other fitted models, the LEGu distribution has the highest log-likelihood and the lowest AIC and BIC values. Figure 6 can be used as support for this fit.

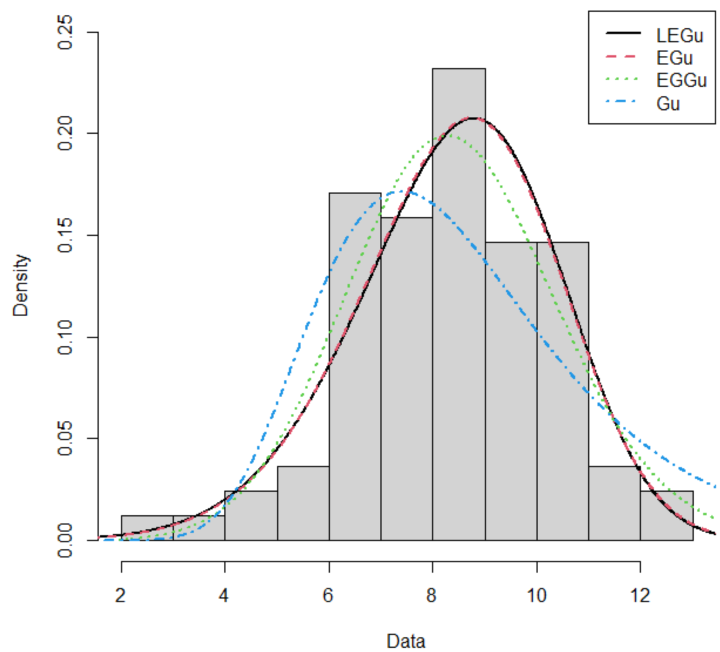

Figure 6: Fitted pdfs for the LEGu distribution with its comparators based on data set.

7. CONCLUSION

This paper developed a novel probability distribution that belongs to the Lindley-G family of distributions called the Lindley Exponentiated Gumbel (LEGu) distribution. Moments, a moment generating function, and dependability assessments are just a few of the significant mathematical aspects we deduced for this recently created model. The maximum likelihood estimation (MLE) method, a widely applied and effective method for parameter estimation in statistical models, was used to estimate the model parameters. The MLE allows us to obtain the most likely values of the parameters that best fit the data to the proposed distribution.

We used metrics such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to evaluate the performance of the newly constructed LEGu distribution with that of existing models. These factors are crucial for choosing a model, and our results show that the LEGu distribution is more reliable and adaptable than the other distributions taken into account.

Overall, the findings show that adding parameters to the distribution increases its resilience and adaptability, enabling it to better suit a variety of data types across multiple baseline distributions and comparators. The LEGu distribution is a useful tool for modelling a variety of data in a variety of applications, making a significant addition to the field of probability distributions.

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DOI: https://doi.org/10.15379/ijmst.v10i2.2935