Nonexistence of Kneser solution of neutral delay difference equation

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Abstract

By creating suitable standards for the nonexistence of so-called Kneser solutions, this study seeks to augment the third order neutral delay difference equations current oscillation results. The results of combining new and previous findings. All of the answers to the studied equations oscillate thanks to our work.

Keywords : Non-oscillation, difference equation, Kneser solution, neutral delay.

MSC:39A10

1. Introduction

In this paper, we look into the third-order neutral delay difference equation's Kneser solution.

$$\Delta\left(a_2(\iota)\Delta\left(a_1(\iota)\Delta y(\iota)\right)\right)+c_1(\iota)x\left(\varsigma(\iota)\right)=0, \iota \geq \iota_0 > 0, \tag{1}$$

where $y(\iota)=x(\iota)+\omega(\iota)x(\mu(\iota))$.

In this paper, we assume the following:

(A1) $\omega(\iota)$ and c_1 are positive real sequences, $0 \leq \omega(\iota) \leq \omega_0 < \infty$ and $c_1(\iota)$ does not vanish identically;

(A2) ς is a real valued increasing sequence with $\varsigma(\iota) < \iota$ and $\lim_{\iota\to\infty}\mu(\iota)=\infty$, μ is a real valued increasing sequence with $\mu(\iota) < \iota$ and $\lim_{\iota\to\infty}\mu(\iota)=\infty$;

(A3) The real sequences are $\{a_1\}$, $\{a_2\}$ which satisfy $\sum_{\iota_0}^{\infty}\frac{1}{a_1(\iota)}=\infty;\sum_{\iota_0}^{\infty}\frac{1}{a_2(\iota)}=\infty;$

(A4) $\mu(\iota) \le \iota$ *and* $\mu_\circ \varsigma = \varsigma_\circ \mu$;

(or)

(A5) $\mu(\iota) > \iota, \Delta\varsigma(\iota) > 0.$

A nontrivial sequence $\{y(\iota)\}$ that satisfy (1) for all $\iota \ge \iota_0$ is what we refer to as a solution of (1). If a solution $\{y(\iota)\}$ is neither ultimately negative nor ultimately positive, it is said to be oscillatory; otherwise, it is said to be nonoscillatory. If all of an equation's solution oscillate, the equation itself is said to be oscillatory. Numerous writers have examined the issue of the oscillation of second order difference equations and have offered a variety of methods for determining the oscillation criterion for the investigated equations, such as [1]-[7].

Over the last two decades, difference equations have been huge attention among researchers [13], [14], [15], [16]. Moreover difference equations have too many applications between many branches of science [8], [9], [10], [11]. There are many examples related to difference equations which are increasing day by day and will continue to increase.

Using the conventional outcomes of [12], [17], [18], [19], we say (1) has property A if any solution x of (1), is either oscillatory or fulfils $\lim_{\iota\to\infty} x(\iota) = 0$.

We denote some operators:

$$F_0 y = y, F_1 y = a_1 \Delta y, F_2 y = a_2 \Delta(a_1 \Delta y), F_3 y = \Delta(a_2 \Delta(a_1 \Delta y)).$$

Kneser Solutions are based on the fundamental presumptions of (A1)-(A5) and solutions x whose matching sequence $y \in B_1$. As a result, it is obvious that (1) has property A if any nonoscillatory solution x is of Kneser type and $\lim_{\iota\to\infty} x(\iota) = 0$.

Lemma 1.1.

Assume that x is a non-oscillatory solution to (1) and that (A1) − (A3) hold. Then there are only two classes that y could belong to:

$$B1 = \{ y(\iota) : \exists \text{ such that } y(\iota) F_1 y(\iota) < 0, y(\iota) F_2 y(\iota) > 0, \forall \iota \ge I \},$$

$$B2 = \{ y(\iota) : \exists \text{ such that } y(\iota) F_1 y(\iota) > 0, y(\iota) F_2 y(\iota) > 0, \forall \iota \ge I \}.$$

Proof. It is omitted as rather easy.

2 Main Results

We use the notation shown below:

$$\chi_k(\iota, u) = \sum_{s=u}^{\iota-1} \frac{1}{a_k(s)},\ k = 1,2,\ \chi(\iota, u) = \sum_{s=u}^{\iota-1} \frac{\chi_2(\iota, s)}{a_\iota(s)} \text{ and}$$

$$C(\iota) = \min\left\{ c_1(\iota), c_1(\mu(\iota)) \right\}, C_1(\iota) = \min\left\{ c_1(\zeta^{-1}(\iota)), c_1((\zeta^{-1}\mu(\iota))) \right\},\ \iota \ge u \ge \iota_0$$

Theorem 2.1

Suppose that (A1) – (A4) hold and if there exists a real sequence ρ(ι) satisfying ζ(ι) < ρ(ι) and $\mu^{-1}(\rho(\iota)) < \iota$ such that the first order delay difference equation

$$\Delta\delta(\iota) + \frac{\mu_0}{\mu_0 + \omega_0} C(\iota) \chi(\rho(\iota), \varsigma(\iota)) \delta(\mu^{-1}(\rho(\iota))) = 0 \tag{2}$$

is oscillatory, then B1 = ϕ.

Proof:

By contradiction, x is a Kneser solution of (1). With no loss of generality, we take $x(\iota), x(\mu(\iota)) > 0, x(\zeta(\iota)) > 0$ for $\iota \ge \iota_1 \ge \iota_0$. This implies that

$$y>0, F_1 y<0, F_2 y>0, F_3 y \le 0 \ on [\iota_1, \infty)$$

By (1), (A2) and (A4), we observe that

$$\begin{aligned} 0 &= \omega_0 \Delta\left(F_2 y\left(\mu(\iota)\right)\right) + \omega_0 c_1\left(\mu(\iota)\right) x\left(\zeta\left(\mu(\iota)\right)\right), \\ &\ge \frac{\omega_0}{\mu_0} \Delta\left(F_2 y\left(\mu(\iota)\right)\right) + \omega_0 c_1\left(\mu(\iota)\right) x\left(\zeta\left(\mu(\iota)\right)\right), \\ &\ge \frac{\omega_0}{\mu_0} \Delta\left(F_2 y\left(\mu(\iota)\right)\right) + \omega_0 c_1\left(\mu(\iota)\right) x\left(\mu\left(\zeta(\iota)\right)\right), \end{aligned}$$

where $\mu_0 > 0$

Adding (1) with the previous inequality, we obtain

$$\begin{aligned} 0 &\ge F_3 y(\iota) + \frac{\omega_0}{\mu_0} \Delta\left(F_2 y\left(\mu(\iota)\right)\right) + \omega_0 c_1\left(\mu(\iota)\right) x\left(\mu\left(\zeta(\iota)\right)\right) + c_1(\iota) x\left(\zeta(\iota)\right) \\ &\ge F_3 y(\iota) + \frac{\omega_0}{\mu_0} \Delta\left(F_2 y\left(\mu(\iota)\right)\right) + C(\iota)\left(\omega_0 x\left(\mu\left(\zeta(\iota)\right) + x\left(\zeta(\iota)\right)\right)\right). \end{aligned} \tag{3}$$

Applying (A1) in the definition of y(ι), we obtain

$$\begin{aligned} y\left(\zeta(\iota)\right) &= x\left(\zeta(\iota)\right) + \omega\left(\zeta(\iota)\right) x\left(\mu\left(\zeta(\iota)\right)\right) \\ &\le x\left(\zeta(\iota)\right) + \omega_0 x\left(\mu\left(\zeta(\iota)\right)\right). \end{aligned}$$

Then (3) becomes

$$F_3 y(\iota) + \frac{\omega_0}{\mu_0} F_3\left(\mu(\iota)\right) + C(\iota) y\left(\zeta(\iota)\right) \le 0.$$

or equivalently,

$$\Delta\left(F_2 y(\iota) + \frac{\omega_0}{\mu_0} F_2\left(\mu(\iota)\right)\right) + C(\iota) y\left(\zeta(\iota)\right) \le 0 \tag{4}$$

Similarly, it is implied by the monotonicity of $F_2 y(\iota)$ that

$$-F_1 y(u) \ge F_1 y(v) - F_1 y(u) = \sum_{s=u}^{v-1} \frac{F_2 y(s)}{a_2(s)} \ge F_2 y(v) \chi_2(v,u), \text{ for } v \ge u \ge \iota_1.$$

Summing above inequality from u to v-1

$$\begin{aligned} y(u) &\ge F_2 y(v) \sum_{s=u}^{v-1} \frac{1}{a_1(s)} \sum_{u}^{v-1} \frac{1}{a_2(s)} \\ &\ge F_2 y(v) \chi(v,u) \end{aligned} \tag{5}$$

For $\rho(\iota) \geq \zeta(\iota) \geq \iota,$ consequently, we have

$$y(\zeta(\iota)) \geq F_2 y(\rho(\iota)) \chi(\rho(\iota), \zeta(\iota))$$

which by reason of (4), gives that

$$\Delta\left(F_2 y(\iota) + \frac{\omega_0}{\mu_0} F_2 y(\mu(\iota))\right) + C(\iota) y(\zeta(\iota)) \leq 0 .$$

$$\Delta\left(F_2 y(\iota) + \frac{\omega_0}{\mu_0} F_2 y(\mu(\iota))\right) + C(\iota) \chi(\rho(\iota), \zeta(\iota)) F_2 y(\rho(\iota)) \leq 0 \qquad (6)$$

Now, we define

$$\delta(\iota) = F_2 y(\iota) + \frac{\omega_0}{\mu_0} F_2 y(\mu(\iota)) > 0$$

By (A4) and the fact that F_2y is nonincreasing, we get

$$\delta(\iota) = F_2 y(\mu(\iota)) \left(1 + \frac{\omega_0}{\mu_0}\right)$$

or equivalently,

$$F_2 y(\rho(\iota)) \geq \left(\frac{\mu_0}{\mu_0 + \omega_0}\right) \delta\left(\mu^{-1}(\rho(\iota))\right) \qquad (7)$$

Applying (7) in (6), we see that ζ is a positive solution of the first order delay difference in equality

$$\Delta\delta(\iota) + \left(\frac{\mu_0}{\mu_0 + \omega_0}\right) \delta\left(\mu^{-1}(\rho(\iota))\right) C(\iota) \chi(\rho(\iota), \zeta(\iota)) \leq 0, \qquad (8)$$

the difference equation (2) also has a positive solution, which is a contradiction, according to the result [[5], Theorem 2.2]. The proof is completed. Therefore, the class B_1 is empty.

Corollary 2.2

Suppose that (A1)−(A4) hold. If there is a sequence $\zeta(\iota) < \rho(\iota)$ and $\mu^{-1}(\rho(\iota)) < \iota,$ such that

$$\lim_{\iota\to\infty} \text{in } f \sum_{\mu^{-1}(\rho(\iota))}^{\iota} C(s)\chi(\rho(s),\zeta(s)) > \frac{\mu_0+\omega_0}{\mu_0 e}, \tag{9}$$

then $B_1 = \phi$.

Theorem 2.3

Suppose that (A1)-(A4) hold. If there exists a sequence $\eta_1(\iota)$ which is real and satisfying $\eta_1(\iota) < \iota$ and $\zeta(\iota) < \mu(\eta_1(\iota))$ such that

$$\lim_{\iota\to\infty} sup\,\chi(\mu(\eta_1(\iota)),\zeta(\iota)) \sum_{\eta_1(\iota)}^{\iota} C(s) > \frac{\mu_0+\omega_0}{\mu_0}, \tag{10}$$

then $B_1 = \phi$.

Proof:

We receive (4) when the theorem 2.1 is proved.

Using the knowledge that y is decreasing and adding up this inequality from $\eta_1(\iota)$ to $\iota-1$, we can see that,

$$\begin{aligned} F_2 y(\eta_1(\iota)) + \frac{\omega_0}{\mu_0} F_2 y(\mu(\eta_1(\iota))) &\geq F_2 y(\iota) + \frac{\omega_0}{\mu_0} F_2 y(\mu(\iota)) + \sum_{\eta_1(\iota)}^{\iota-1} C(s) y(\zeta(s)) \\ &\geq \sum_{\eta_1(\iota)}^{\iota-1} C(s) y(\zeta(s)) \\ &\geq y(\zeta(\iota)) \sum_{\eta_1(\iota)}^{\iota-1} C(s) \end{aligned} \tag{11}$$

Since $\mu(\eta_1(\iota)) < \mu(\iota)$ and $F_2 y(\iota)$ is nonincreasing, we have

$$F_2 y(\mu(\eta_1(\iota))) \geq F_2 y(\mu(\iota))$$

Therefore, (11) becomes

$$F_2 y(\mu(\eta_1(\iota)))\left(1+\frac{\omega_0}{\mu_0}\right) \geq y(\zeta(\iota)) \sum_{\eta_1(\iota)}^{\iota-1} C(s) \tag{12}$$

Applying (5), with the conditions $u = \zeta(\iota),\ v = \mu(\eta_1(\iota)),$ and in (17), we see that

$$F_2 y\left(\mu\left(\eta_1(\iota)\right)\right)\left(1+\frac{\omega_0}{\mu_0}\right) \geq F_2 y\left(\mu\left(\eta_1(\iota)\right)\right)\chi\left(\mu\left(\eta_1(\iota),\zeta(\iota)\right)\right)\sum_{\eta_1(\iota)}^{\iota-1} C(s)$$

$$\frac{\mu_0+\omega_0}{\mu_0} \geq \chi\left(\mu\left(\eta_1(\iota)\right),\zeta(\iota)\right)\sum_{\eta_1(\iota)}^{\iota-1} C(s),$$

selecting the limit supremum on both sides of the aforementioned inequality, which is against (10). The evidence is conclusive.

Corollary 2.4

Assume (A1)−(A4) and $\zeta(\iota) < \mu\left(\mu(\iota)\right)$. If

$$\limsup_{\iota\to\infty} \chi\left(\mu\left(\mu(\iota)\right),\zeta(\iota)\right)\sum_{\mu(\iota)}^{\iota} C(s) > \frac{\mu_0+\omega_0}{\mu_0}, \tag{13}$$

then $B_1=\phi$.

Theorem 2.5

Assume that (A1) - (A3), (A5) and $\zeta\left(\mu(\iota)\right) < \iota$. If the first order delay difference equation

$$\Delta\delta(\iota) + \left(\frac{\zeta_0\mu_0}{\mu_0+\omega_0}\right) C_1(\iota)\chi\left(\mu(\iota),\iota\right)\delta\left(\zeta\left(\mu(\iota)\right)\right) = 0 \tag{14}$$

is oscillatory, then $B_1=\phi$.

Proof : By the contradiction that x is a Kneser solution of (1), with no lose of generality. We take $x(\iota),\ x\left(\mu(\iota)\right) > 0,\ x\left(\zeta(\iota)\right) > 0$ for $\iota \geq \iota_1 \geq \iota_0$, which implies that

$$y > 0, F_1 y < 0, F_2 y > 0, F_3 y \leq 0,\ on\ [\iota_1,\infty).$$

By (1) and (A2), (A5), we obtain

$$\begin{aligned}
0 &= \Delta\left(F_2 y(\iota)\right) + c_1(\iota)x\left(\zeta(\iota)\right)\\
&> \Delta\left(\mathrm{F}_2\mathrm{y}(\iota)\right) + c_1(\iota)x(\iota)\\
&> \Delta\left(\mathrm{F}_2\mathrm{y}\left(\zeta^{-1}(\iota)\right)\right) + c_1\left(\zeta^{-1}(\iota)\right)x(\iota)\\
&> \frac{1}{\zeta_0}\Delta\left(F_2 y\left(\zeta^{-1}(\iota)\right)\right) + c_1\left(\zeta^{-1}(\iota)\right)x(\iota)
\end{aligned}$$

where $\zeta_0 > 0$ and similarly,

$$0=\omega_0\Delta\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\Delta\left(F_2y\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)\right)\right)+\omega_0c_1\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)x\left(\mu\left(\iota\right)\right)$$
$$\geq\frac{\omega_0}{\zeta_0\mu_0}\Delta\left(F_2y\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)\right)+\omega_0c_1\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)x\left(\mu\left(\iota\right)\right).$$

Combining the above inequalities, gives that

$$\frac{1}{\zeta_0}\Delta\left(F_2y\left(\zeta^{-1}\left(\iota\right)\right)\right)+\frac{\omega_0}{\zeta_0\mu_0}\Delta\left(F_2y\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)\right)+c_1\left(\zeta^{-1}\left(\iota\right)\right)x\left(\iota\right)+\omega_0c_1\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)x\left(\mu\left(\iota\right)\right)\leq 0,$$

$$\Delta\left(\frac{F_2y\left(\zeta^{-1}\left(\iota\right)\right)}{\zeta_0}+\frac{\omega_0}{\zeta_0\mu_0}\left(F_2y\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)\right)\right)+C_1\left(\iota\right)y\left(\iota\right)\leq 0. \tag{15}$$

Now, we Fix

$$\delta\left(\iota\right)=\frac{F_2y\left(\zeta^{-1}\left(\iota\right)\right)}{\zeta_0}+\frac{\omega_0}{\zeta_0\mu_0}F_2y\left(\zeta^{-1}\left(\mu\left(\iota\right)\right)\right)>0.$$

By the assumption of (A5) and the fact that $F_2y\left(\iota\right)$ is nonincreasing, it can easily see that

$$\delta\left(\iota\right)\leq\frac{F_2y\left(\zeta^{-1}\left(\iota\right)\right)}{\zeta_0}\left(1+\frac{\omega_0}{\mu_0}\right)$$
$$F_2y\left(\mu\left(\zeta\left(\zeta^{-1}\left(\iota\right)\right)\right)\right)\geq\delta\left(\mu\left(\zeta\left(\iota\right)\right)\right)\left(\frac{\mu_0\zeta_0}{\mu_0+\omega_0}\right)$$
$$F_2y\left(\mu\left(\iota\right)\right)\geq\delta\left(\zeta\left(\mu\left(\iota\right)\right)\right)\left(\frac{\mu_0\zeta_0}{\mu_0+\omega_0}\right) \tag{16}$$

From (5) with $v=\mu\left(\iota\right),\quad u=\iota$ in (16), we have

$$y\left(\iota\right)\geq F_2y\left(\mu\left(\iota\right)\right)\chi\left(\mu\left(\iota\right),\iota\right)$$
$$\geq\delta\left(\zeta\left(\mu\left(\iota\right)\right)\right)\chi\left(\mu\left(\iota\right),\iota\right)\left(\frac{\zeta_0\omega_0}{\mu_0+\omega_0}\right).$$

Applying the definition of ς and the above inequality in (15), we get

$$\Delta\delta\left(\iota\right)+\left(\frac{\zeta_0\mu_0}{\mu_0+\omega_0}\right)C_1\left(\iota\right)\chi\left(\mu\left(\iota\right),\iota\right)\delta\left(\zeta\left(\mu\left(\iota\right)\right)\right)\leq 0.$$

The difference equation (14) also has a positive solution, which is a contradiction, according to the result [[5], Theorem 2.2]. The proof is completed. Therefore, the class B_1 is empty.

Corollary 2.6

Suppose that (A1)-(A3), (A5) hold and if

$$\liminf_{\iota\to\infty} \sum_{\zeta(\mu(\iota))}^{\iota-1} C_1(s)\chi(\mu(s),s) > \frac{\mu_0+\omega_0}{\zeta_0\mu_0}, \tag{17}$$

then $B_1=\phi$.

Theorem 2.7:

Assume that for (1), class $B_2=\phi$. (1) is oscillatory if all the conditions of Theorem 2.1, Theorem 2.3, or Theorem 2.5 are met.

3 Example

Consider the following system

$$\Delta\left(\frac{1}{\iota}\Delta\left(\frac{1}{\iota^2}\Delta\left(x(\iota)+2\iota x(3\iota)\right)\right)\right)+\frac{1}{\iota+1}x\left(\frac{1}{2\iota}\right)=0,\ \iota\ge 1. \tag{18}$$

Here $a_2(\iota)=\frac{1}{\iota}, a_1(\iota)=\frac{1}{\iota^2}, \omega(\iota)=2\iota\ge 0, \mu(\iota)=\iota\le\iota, c_1(\iota)=\frac{1}{\iota+1}, \zeta(\iota)=\frac{\iota}{2}<\iota$. Furthermore, taking $\rho(\iota)=2\iota, \eta_1(\iota)=\frac{1}{4\iota^2}$ implies that $\mu^{-1}(\rho(\iota))=\frac{1}{6\iota}<\iota$ and $\zeta(\iota)=\frac{\iota}{2}<2\iota.$ So $\zeta(\iota)<\rho(\iota)$ and $\sum_{s=\iota_0}^{\infty}\frac{1}{a(s)}=\sum_{s=\iota_0}^{\infty}\iota^2=\infty, \sum_{s=\iota_0}^{\infty}\frac{1}{a_2(s)}=\sum_{s=\iota_0}^{\infty}\iota=\infty.$ Also $\mu(\eta_1(\iota))=\frac{3}{4\iota^2}>\zeta(\iota)$. From these values, we cleared all the assumptions of theorem (2.1), theorem (2.3) or theorem (2.5) are fulfilled. Hence above (18) is oscillatory.

4. Conclusion

This study's findings serve to generalise the oscillatory results for (1) and are then paired with pre-existing criteria to weed out solutions from the class B2 that are founded on either (A4) or (A5) assumptions. Moreover the monotonic properties of solutions presented in the Theorems that can be used in various methods, viz, comparison principle and summation averaging technique, etc, are applied in the theory of oscillation.

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