

A New Version of Prakaamy Distribution with Properties and Applications

D. Vedavathi Saraja^{1*}, C. Subramanian², N. Srinivasa Rao³

^{1,2}Department of Statistics, Annamalai University, Annamalainagar, Tamil Nadu, India

³Department of Statistics, Andhra Loyola College, Vijayawada, India

^{1,*}Email: sarajayoganand@gmail.com, ²Email: manistat@yahoo.co.in, ³Email: nunnasr@gmail.com

Abstract: This paper introduces a novel statistical model, the Length-Biased version of the Prakaamy distribution, characterized by one parameter. The study presents a comprehensive analysis of this distribution, including the derivation of its probability density function (PDF), cumulative distribution function (CDF), moments, moment generating function, characteristics function, reliability analysis, ordered statistics, maximum likelihood estimation of parameter, entropies, likelihood ratio test, and Bonferroni and Lorenz curves. To validate the effectiveness of the proposed model, it is applied to two real-life time data sets, demonstrating its applicability and utility in practical scenarios.

Keywords: Length-Biased distribution, Prakaamy distribution, reliability analysis, order statistics, maximum likelihood estimators.

1. INTRODUCTION

Weighted distributions are a statistical concept used to model situations where recorded observations cannot be assumed to be randomly sampled from the actual underlying distribution. This can occur for various reasons, such as damaged or incomplete data, or when events are observed in a non-random or non-observable manner. In such cases, the resulting values may not accurately reflect the true distribution, and units or events may not have equal chances of occurrence as they would if they followed the exact distribution. Weighted distributions are applied in various research areas, including biomedicine, reliability analysis, ecology, and branching processes. The concept of weighted distributions was first introduced by Sir Ronald A. Fisher in 1934 to address the issue of ascertainment bias. Later, C.R. Rao, a renowned statistician, developed this concept in a more unified manner. Rao's work involved modelling statistical data when standard distributions were not suitable due to unequal probabilities in recording observations. Weighted models were then formulated to account for this bias. Weighted distributions can take different forms depending on the nature of the bias. For instance, a weighted distribution reduces to a length-biased distribution when the weight function considers only the length or size of the units. The concept of length-biased sampling was introduced by David R. Cox in 1969 and Marvin Zelen in 1974. More generally, when the sampling mechanism selects units with a probability proportional to some measure of the unit size, the resulting distribution is called size-biased. There are various good sources which provide the detailed description of weighted distributions. Different authors have reviewed and studied the various weighted probability models and illustrated their applications in different fields. Weighted distributions are applied in various research areas related to reliability, biomedicine, ecology and branching processes. Afaq et al (2016) have obtained the length biased weighted version of lomax distribution with properties and applications. Reyad et al. (2017), obtained the length biased weighted frechet distribution with properties and estimation. Khan et al. (2018) discussed the weighted modified Weibull distribution. Rather et al (2018) obtained a new size biased Ailamujia distribution with applications in engineering and medical science which shows more flexibility than classical distributions. Subramanian and Rather (2018) studied the weighted exponentiated Mukherjee-Islam distribution and its statistical properties. Mudasir and Ahmad (2018), discussed the characterization and estimation of length biased Nakagami distribution. Modi and Gill (2015), discussed the length biased weighted Maxwell distribution. Dey et al (2015) discussed weighted exponential distribution with its properties and different methods of estimation. Kilany (2016) have obtained the weighted version of lomax distribution. Recently, Rather and Subramanian (2019), discussed the length biased erlang truncated exponential distribution with applications to real life data which shows more flexibility than classical distributions. Shenbagaraja et al. (2019), discussed on length biased Garima distribution which shows more flexibility than the classical distributions. Mathew and Chesneau (2020), discussed on Marshall-olkin length biased Maxwell distribution and its applications.

2. LENGTH BIASED PRAKAAMY (LBP) DISTRIBUTION

The probability density function (PDF) of Prakaamy distribution is given by

$$g(y; \theta) = \frac{\theta^6}{\theta^5 + 120} (1 + y^5) e^{-\theta y} \quad ; y > 0, \theta > 0 \quad (01)$$

The corresponding cumulative distribution function (CDF) of Prakaamy distribution is

$$G(y; \theta) = 1 - \left(1 + \frac{\theta y (\theta^4 y^4 + 5\theta^3 y^3 + 20\theta^2 y^2 + 60\theta y + 120)}{\theta^5 + 120} \right) e^{-\theta y} \quad y > 0, \theta > 0 \quad (02)$$

Let Y be a non-negative random variable with PDF $g(y; \theta)$, then the PDF of the weighted random variable Y_w is called the weighted distribution which is given by

$$f(y; \theta, c) = \frac{w(y) g(y; \theta)}{E(w(y))}$$

where $w(y)$ be a non-negative weight function and $E(w(y)) = \int_0^\infty w(y) g(y; \theta) dy < \infty$. Note that different choices of the weight function $w(y)$ gave different weighted distributions. Consequently, for $w(y) = y$ the resulting distribution is called length-biased distribution with the PDF given by

$$f(y; \theta) = \frac{y g(y; \theta)}{E(y)} \quad (03) \quad \text{Now}$$

$$E(y) = \int_0^\infty y g(y; \theta) dy$$

$$E(y) = \int_0^\infty y \frac{\theta^6}{\theta^5 + 120} (1 + y^5) e^{-\theta y} dy$$

$$E(y) = \frac{\theta^6}{\theta^5 + 120} \left(\int_0^\infty e^{-\theta y} y dy + \int_0^\infty e^{-\theta y} y^6 dy \right)$$

$$E(y) = \frac{\theta^6}{\theta^5 + 120} \left(\frac{1}{\theta^2} + \frac{6!}{\theta^7} \right)$$

After simplification we get

$$E(y) = \frac{\theta^6}{\theta^5 + 120} \frac{1}{\theta^2} \left(1 + \frac{6!}{\theta^5} \right)$$

$$E(y) = \frac{\theta^6}{\theta^5 + 120} \frac{1}{\theta^7} (\theta^5 + 6!)$$

$$E(y) = \frac{1}{(\theta^5 + 120)\theta} (\theta^5 + 6!) \quad (04)$$

Substituting equation (01) and (04) in (03) we get

$$f(y; \theta,) = \frac{\theta^7 y (1 + y^5) e^{-\theta y}}{(\theta^5 + 6!)} \quad (05)$$

The corresponding CDF of Length-biased Prakaamy distribution is

$$F(y; \theta) = \int_0^y \frac{\theta^7 y(1 + y^5) e^{-\theta y}}{(\theta^5 + 6!)} dy$$

$$F(y; \theta) = \frac{\theta^7}{(\theta^5 + 6!)} \left(\int_0^y e^{-\theta y} y dy + \int_0^y e^{-\theta y} y^6 dy \right)$$

After simplification we get

$$F(y; \theta) = \frac{\theta^5 \gamma(2, \theta y) + \gamma((7, \theta y))}{(\theta^5 + 6!)} \tag{06}$$

Graphical representation of PDF and CDF plots of length-biased Prakaamy distribution is shown in Figure 1 and Figure 2 respectively as below:

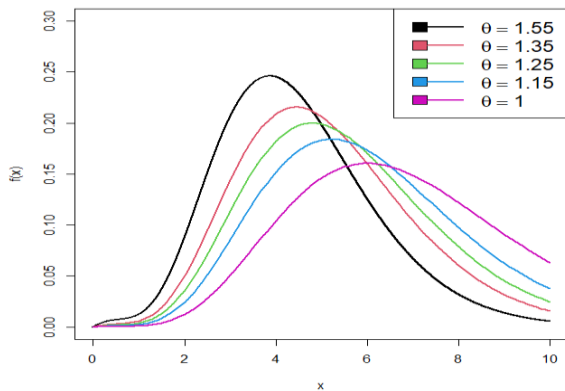


Figure 1: PDF plot of LBP distribution

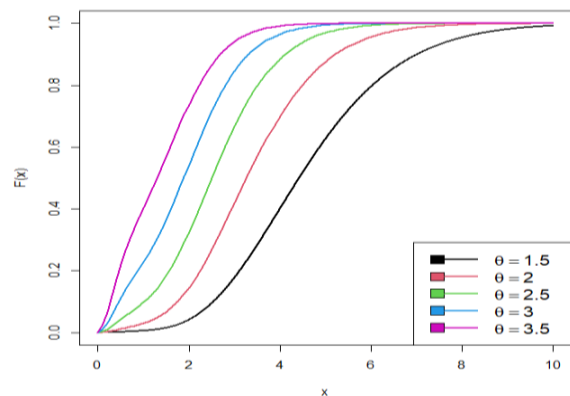


Figure 2: CDF plot of LBP distribution

3. MOMENTS OF LENGTH-BIASED PRAKAAMY DISTRIBUTION

Let Y denotes the random variable following Length-biased Prakaamy distribution with parameters θ , then the r th raw moment about origin of the Length-biased Prakaamy distribution is given by

$$\mu'_r = \int_0^\infty y^r f(y; \theta) dy$$

$$\mu'_r = \int_0^\infty y^r \frac{\theta^7 y(1 + y^5) e^{-\theta y}}{(\theta^5 + 6!)} dy$$

After simplification we get

$$\mu'_r = \frac{(r + 1)! (\theta^5 + (r + 6)(r + 5)(r + 4)(r + 3)(r + 2))}{\theta^{r+1} (\theta^5 + 6!)}$$

4. MOMENT GENERATING FUNCTION AND CHARACTERISTICS FUNCTION

The moment generating function of Length-biased Prakaamy distribution is given by

$$M_{Y_L}(t) = \int_0^{\infty} e^{ty} f(y; \theta) dy$$

$$M_{Y_L}(t) = \int_0^{\infty} e^{tx} \frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5 + 6!)} dy$$

After simplification we get

$$M_{Y_L}(t) = \sum_{j=0}^{\infty} \frac{(t)^j (j+1)! (\theta^5 + (j+6)(j+5)(j+4)(j+3)(j+2))}{j! \theta^{j+1} (\theta^5 + 6!)}$$

Also, the characteristics function of Length-biased Prakaamy distribution is given by

$$\phi_{Y_L}(t) = \sum_{j=0}^{\infty} \frac{(it)^j (j+1)! (\theta^5 + (j+6)(j+5)(j+4)(j+3)(j+2))}{j! \theta^{j+1} (\theta^5 + 6!)}$$

5. THE RELIABILITY ANALYSIS

In this section, we have obtained the survival function, hazard rate and Reverse hazard rate function of the proposed Length-biased Prakaamy distribution.

5.1. Survival function

The survival function, also known as the reliability function, is a fundamental concept in probability theory and statistics, especially in the context of survival analysis. It is used to model and analyse the time to an event, such as the failure of a system, the lifespan of a product, or the time until a patient's recovery or death. The survival function provides valuable information about the likelihood that a system or entity will continue to operate or exist beyond a specified time. The reliability function or the survival function of Length-biased Prakaamy distribution can be computed as

$$S(y_L) = 1 - F(y; \theta)$$

$$S(y_L) = 1 - \frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5 + 6!)}$$

5.2. Hazard function

The hazard function, also known as the hazard rate, is a crucial concept in survival analysis and reliability engineering. It represents the instantaneous failure rate or the force of mortality, which is the likelihood of an event (such as system failure, death, or any other event of interest) occurring at a specific point in time, given that it has not happened up to that time. The hazard function of Length-biased Prakaamy distribution is given by

$$h(y_L) = \frac{f(y; \theta)}{1 - F(y; \theta)}$$

$$h(y_L) = \frac{\left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5 + 6!)} \right)}{\left(1 - \frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5 + 6!)} \right)}$$

$$h(y_L) = \frac{(\theta^7 y(1+y^5)e^{-\theta y})}{((\theta^5 + 6!) - (\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)))}$$

Graphical representation of survival function and hazard function plots of length-biased Prakaamy distribution is shown in Figure 3 and Figure 4 respectively as below:

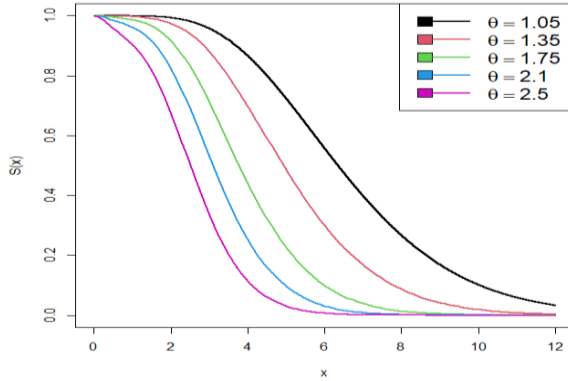


Figure 3: Survival function plot of LBP distribution

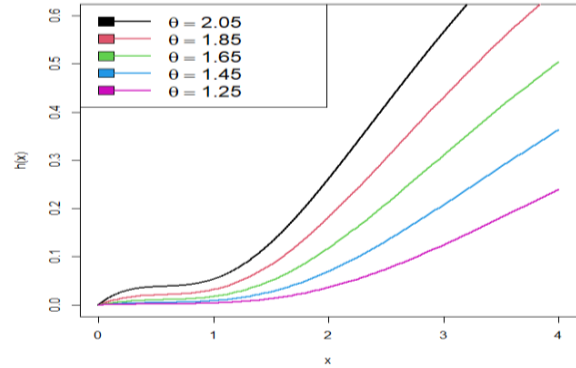


Figure 4: Hazard function plot of LBP distribution

5.3. Reverse Hazard Function

The reverse hazard function of Length-biased Prakaamy distribution is given by

$$h^r(y_L) = \frac{f(y; \theta)}{F(y; \theta)}$$

$$h^r(y_w) = \frac{\left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5 + 6!)} \right)}{\left(\frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5 + 6!)} \right)}$$

$$h^r(y_w) = \frac{\theta^7 y(1+y^5)e^{-\theta y}}{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}$$

6. ORDERED STATISTICS

Order statistics are a set of values obtained by arranging a sample of random variables in ascending order. They provide insights into the distribution of the sample, including the minimum and maximum values, as well as percentiles like the median. Order statistics are essential in statistical analysis and non-parametric statistics. Let $Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ be a random sample of size 'n' drawn from a given population following Length-biased Prakaamy distribution. Then the ordered statistics associated with the given sample is given by

$$Y_{(1)} \leq Y_{(2)} \leq Y_{(3)} \leq \dots \leq Y_{(n)}$$

Where,

$$Y_{(1)} = \min(Y_1, Y_2, \dots, Y_n)$$

And

$$Y_{(n)} = \max(Y_1, Y_2, \dots, Y_n)$$

Now, the PDF of r th ordered statistics $Y_{(r)}$ is

$$f_{Y_{(r)}}(y) = \frac{n!}{(r-1)!(n-r)!} f(y)(F(y))^{r-1}(1-F(y))^{n-r}$$

$$f_{Y_{(r)}}(y) = \frac{n!}{(r-1)!(n-r)!} \left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5+6!)} \right) \left(\frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5+6!)} \right)^{r-1}$$

$$\times \left(1 - \frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5+6!)} \right)^{n-r}$$

Therefore, the PDF of highest ordered statistics $Y_{(n)}$ is

$$f_{Y_{(n)}}(y) = n \left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5+6!)} \right) \left(\frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5+6!)} \right)^{n-1}$$

And the PDF of first ordered statistics $Y_{(1)}$ is

$$f_{Y_{(1)}}(y) = n \left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5+6!)} \right) \left(1 - \frac{\theta^5 \gamma(2, \theta y) + \gamma(7, \theta y)}{(\theta^5+6!)} \right)^{n-1}$$

7. ESTIMATION

Maximum Likelihood Estimation (MLE) is a powerful statistical technique for estimating distribution parameters. It involves finding parameter values that maximize the likelihood of observed data given a specific statistical model. MLE offers consistent, asymptotically unbiased, and efficient estimates, making it a widely used tool in fields like economics, biology, and engineering. Let $Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ be the random sample of size n drawn from the Length-biased Prakaamy distribution. Then the likelihood function of the given random sample is given by

$$L(\theta) = \prod_{i=1}^n f(y_i; \theta)$$

$$L(\theta) = \prod_{i=1}^n \frac{\theta^7 y_i(1+y_i^5)e^{-\theta y_i}}{(\theta^5+6!)}$$

$$L(\theta) = \left(\frac{\theta^7}{(\theta^5+6!)} \right)^n \prod_{i=1}^n y_i(1+y_i^5)e^{-\theta y_i}$$

Applying log on both sides, we get

$$\log L(\theta) = \log \left(\left(\frac{\theta^7}{(\theta^5+6!)} \right)^n \prod_{i=1}^n y_i(1+y_i^5)e^{-\theta y_i} \right)$$

$$\log(L(\theta)) = 7n \log(\theta) - n \log(\theta^5+6!) +$$

$$\left(\sum_{i=1}^n \log y_i + \sum_{i=1}^n \log(1+y_i^5) \right) + \left(-\theta \sum_{i=1}^n y_i \right) \tag{07}$$

Differentiating equation (07) partially with respect to θ and equating to zero we get we get

$$\frac{7n}{\theta} - \frac{5\theta^4 n}{\theta^5+6!} - \sum_{i=1}^n y_i = 0$$

$$\frac{7n}{\theta} - \frac{5\theta^4 n}{\theta^5 + 6!} - \sum_{i=1}^n y_i = 0 \tag{08}$$

By solving equation (08) we get the maximum likelihood estimator of the parameter of Length-biased Prakaamy distribution. Since it is very complicated to estimate θ so we use Mathematica or Newton Raphson method.

8. ENTROPY

Entropy, a versatile concept, finds applications in multiple domains. It quantifies disorder, uncertainty, and diversity in systems. In statistics, it gauges the unpredictability of random variables. In economics, it aids the analysis of market dynamics. Entropy serves as a unifying measure for understanding information and randomness across these diverse fields.

8.1. Renyi Entropy

The Rényi entropy is a vital concept in ecology and statistics, often serving as a diversity index. Introduced by Alfréd Rényi in 1957, it provides a versatile way to quantify the diversity or heterogeneity of a dataset. By adjusting a parameter called the order, it can capture different aspects of diversity, making it a valuable tool for studying ecosystems, populations, and probability distributions in various scientific disciplines. The Renyi entropy is defined as

$$e(\beta) = \frac{1}{1-\beta} \log \left(\int_0^\infty (f(y;\theta))^\beta dy \right)$$

$$e(\beta) = \frac{1}{1-\beta} \log \left(\int_0^\infty \left(\frac{\theta^7 y(1+y^5) e^{-\theta y}}{(\theta^5 + 6!)} \right)^\beta dy \right)$$

$$e(\beta) = \frac{1}{1-\beta} \log \left(\left(\frac{\theta^7}{(\theta^5 + 6!)} \right)^\beta \int_0^\infty y^\beta (1+y^5)^\beta e^{-\beta\theta y} dy \right)$$

$$e(\beta) = \frac{1}{1-\beta} \log \left(\left(\frac{\theta^7}{(\theta^5 + 6!)} \right)^\beta \left(\int_0^\infty y^\beta (1+y^5)^\beta e^{-\beta\theta y} dy \right) \right)$$

After simplification we get

$$e(\beta) = \frac{1}{1-\beta} \log \left(\left(\frac{\theta^7}{(\theta^5 + 6!)} \right)^\beta \left(\sum_{k=0}^{\beta} {}^\beta C_k \frac{\Gamma(\beta + 5k + 1)}{(\beta\theta)^{\beta + 5k + 1}} \right) \right)$$

8.2. Tsallis entropy

Tsallis entropy, introduced by Constantino Tsallis in 1988, is a mathematical expression used to characterize systems with non-extensive properties. It is particularly valuable for systems displaying features like long-range interactions, self-organization, and non-Gaussian statistics. Tsallis entropy generalizes the traditional Boltzmann-Gibbs entropy and provides a broader framework for understanding complex systems that do not adhere to standard statistical mechanics. It has applications in fields such as physics, astrophysics, and economics where conventional entropy measures may not capture the system's behaviour adequately.

The Tsallis entropy is given by

$$S_\lambda = \frac{1}{\lambda-1} \left(1 - \int_0^\infty (f(y;\theta))^\lambda dy \right)$$

$$S_\lambda = \frac{1}{\lambda-1} \left(1 - \int_0^\infty \left(\frac{\theta^7 y(1+y^5)e^{-\theta y}}{(\theta^5+6!)} \right)^\lambda dy \right)$$

$$S_\lambda = \frac{1}{\lambda-1} \left(1 - \left(\frac{\theta^7}{(\theta^5+6!)} \right)^\lambda \int_0^\infty (y(1+y^5)e^{-\theta y})^\lambda dy \right)$$

After simplification we get

$$S_\lambda = \frac{1}{\lambda-1} \left(1 - \left(\frac{\theta^7}{(\theta^5+6!)} \right)^\lambda \left(\sum_{k=0}^{\lambda} {}^\lambda C_k \frac{\Gamma(\lambda+5k+1)}{(\lambda\theta)^{\lambda+5k+1}} \right) \right)$$

9. LIKELIHOOD RATIO TEST

Let $Y_1, Y_2, Y_3, Y_4, \dots, Y_n$ be a random sample of size 'n' drawn from a given population following Length-biased Prakaamy distribution .

$$H_0 : f(y) = g(y; \theta) \quad \text{against} \quad H_1 : f(y) = f(y; \theta)$$

In order to test whether the random sample of size n comes from the Prakaamy distribution or Length-biased Prakaamy distribution, then following the likelihood ratio test statistic is used

$$LR = \frac{L_0}{L_1} = \prod_{i=1}^n \frac{g(y_i; \theta)}{f(y_i; \theta)}$$

$$LR = \prod_{i=1}^n \frac{\frac{\theta^6}{\theta^5+120} (1+y_i^5)e^{-\theta y_i}}{\frac{\theta^7 y_i(1+y_i^5)e^{-\theta y_i}}{(\theta^5+6!)}}$$

$$LR = \frac{(\theta^5+6!)^n}{(\theta^5+120)^n \theta^n} \prod_{i=1}^n \left(\frac{1}{y_i} \right)$$

We reject the null hypothesis if the likelihood ratio is small i.e.,

$$LR \leq k$$

Where k is a constant such that

$$P(LR \leq k) = \alpha$$

$$\alpha = P \left(\frac{(\theta^5+6!)^n}{(\theta^5+120)^n \theta^n} \prod_{i=1}^n \left(\frac{1}{y_i} \right) \leq k \right)$$

$$\alpha = P\left(\prod_{i=1}^n \left(\frac{1}{y_i}\right) \leq \frac{k(\theta^5 + 120)^n \theta^n}{(\theta^5 + 6!)^n}\right)$$

10. BONFERRONI AND LORENZ CURVES

Both the Bonferroni and Lorenz curves are highly versatile tools applied in diverse fields, including economics, statistics, reliability, medicine, insurance, and demography. The Bonferroni correction is essential for managing Type I errors in statistical analysis, ensuring rigorous hypothesis testing. Lorenz curves, on the other hand, offer a visual representation of income and wealth disparities, making them crucial for assessing social inequalities and guiding policy decisions. These tools play a pivotal role in various disciplines, aiding in informed decision-making and data analysis.

10.1. Bonferroni Curves

The Bonferroni curve is given by

$$B(p) = \frac{1}{p \mu'_1} \int_0^q y f(y; \theta) dy$$

Where $q = F^{-1}(p)$ and

μ'_1 is the mean of Weighted Prakamy distribution

$$B(p) = \frac{1}{p \mu'_1} \int_0^q y \frac{\theta^7 y(1 + y^5) e^{-\theta y}}{(\theta^5 + 6!)} dy$$

$$B(p) = \frac{1}{p \mu'_1} \frac{\theta^7}{(\theta^5 + 6!)} \int_0^q y^2 (1 + y^5) e^{-\theta y} dy$$

$$B(p) = \frac{1}{p \mu'_1} \frac{\theta^7}{(\theta^5 + 6!)} \left(\int_0^q e^{-\theta y} y^2 dy + \int_0^q e^{-\theta y} y^7 dy \right)$$

After simplification we get

$$B(p) = \frac{1}{p \mu'_1} \frac{\theta^7}{(\theta^5 + 6!)} \left(\frac{1}{\theta^3} \gamma(3, \theta q) + \frac{1}{\theta^8} \gamma(8, \theta q) \right)$$

10.2. Lorenz Curves

The Lorenz curve is given by

$$L(p) = p B(p)$$

$$L(p) = \frac{1}{\mu'_1} \frac{\theta^7}{(\theta^5 + 6!)} \left(\frac{1}{\theta^3} \gamma(3, \theta q) + \frac{1}{\theta^8} \gamma(8, \theta q) \right)$$

11. SIMULATION

Simulations are a powerful tool in statistics and data analysis, and they indeed offer numerous benefits when it comes to understanding the behaviour of maximum likelihood estimators (MLEs) across different sample sizes. Simulations are versatile and can be applied to a wide range of statistical problems. They are not limited to a specific field and can be adapted to various areas such as finance, healthcare, engineering, and more. This adaptability makes

simulations a valuable tool for researchers and decision-makers across different industries. Simulations enable you to test the efficiency and reliability of MLEs under different conditions. This information can be critical when choosing appropriate statistical methods for a particular dataset. It ensures that the chosen estimator performs well and provides reliable results, which is especially important in fields like healthcare and finance where decisions can have significant consequences. Simulations can help you understand how the bias, variance, and efficiency of MLEs change as sample size varies. This information is crucial for selecting an appropriate sample size for your specific application. For instance, it can help you strike a balance between precision (low bias and variance) and cost (collecting larger samples). The inverse CDF technique was employed for data simulation in the R-software and the process was repeated 600 times to calculate bias, variance and mean squared error (MSE). From table 1, it is noted that for different values of parameters with different sample sizes of Length-biased Prakaamy distribution, the decreasing trend has been observed in variance, bias and MSE as the sample size increases. The decreasing bias suggests that the ML estimation tend to approach the true parameter values as the sample size increases. The decreasing variance implies that the estimators become more precise and stable with larger sample sizes, as they exhibit less variability across repeated simulations. Further, Figure 5, Figure 6, Figure 7, Figure 8, Figure 9, and Figure 10 shows the histogram of simulation on different values of the parameters with different the sample size. Consequently, MSE which combines the bias and variance also decreases as the sample size an increase, indicating improved overall estimation accuracy. This result indicates that the performance of ML estimators improves consistently with larger sample sizes in Length-biased Prakaamy distribution.

Table 1: Estimation of Bias, Variance and MSE for different sample sizes

n	$\theta=20$			$\theta=15$		
	Bias	Variance	MSE	Bias	Variance	MSE
25	1.113678	4.96875	6.209029	1.734205	7.197125	10.20459
50	0.4153201	2.525757	2.698248	0.9717218	4.331348	5.275591
75	0.3339129	1.637916	1.749413	0.517613	1.359283	1.627206
100	0.08548331	1.523457	1.530764	0.3129216	0.2885902	0.3865101
200	-0.05538118	0.890632	0.8936991	0.09836069	0.1811009	0.1907758
300	-0.2155352	0.772372	0.8188274	-0.1573127	0.1528819	0.1776292
n	$\theta=11$			$\theta=9$		
	Bias	Variance	MSE	Bias	Variance	MSE
25	1.191601	1.341698	2.76161	0.887049	1.815196	2.602052
50	0.6033993	0.9848901	1.348981	0.7722598	0.9976571	1.594042
75	0.2774067	0.4507087	0.5276632	0.4371463	0.4044518	0.5955487
100	0.3545229	0.3520333	0.4777198	0.1716451	0.3031752	0.3326372
200	0.03971542	0.2354966	0.2370739	-0.06880265	0.1168963	0.1216301
300	-0.05804382	0.192803	0.1961721	-0.1438987	0.01828421	0.03899105
n	$\theta=5$			$\theta=2$		
	Bias	Variance	MSE	Bias	Variance	MSE
25	0.261432	0.298728	0.3670747	0.1292878	0.01568528	0.0324006

50	0.3254438	0.5643284	0.6702421	0.01274988	0.00742706	0.00758962
75	0.2803734	0.5540099	0.6326191	-0.01453137	0.006171473	0.006382634
100	0.2355682	0.3986065	0.4540988	-0.02551115	0.0050785	0.005092088
200	-0.04067243	0.02103617	0.02374435	-0.03686144	0.002977231	0.003628049
300	-0.05204017	0.01055605	0.01221029	-0.00727495	0.001311165	0.00136409

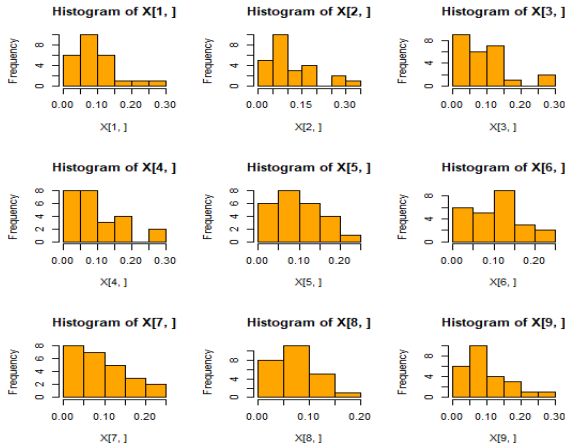


Figure 5: Histogram of simulation when $n=25$,

$\theta=20$

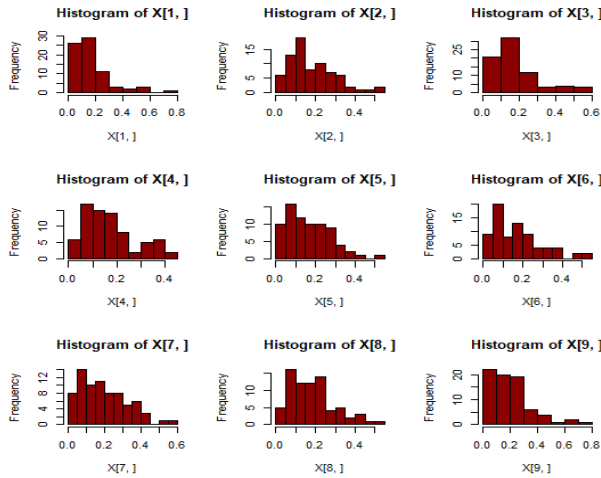


Figure 7: Histogram of simulation when $n=75$, $\theta=11$

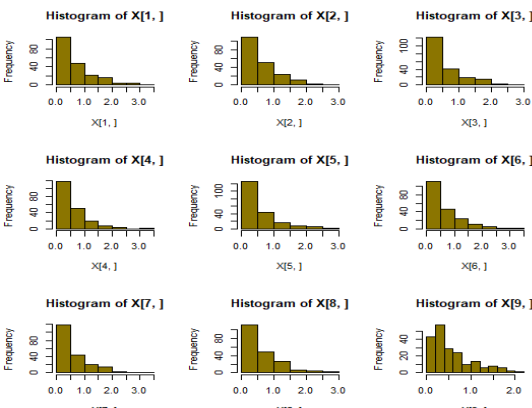


Figure 9: Histogram of simulation when $n=200$, $\theta=5$

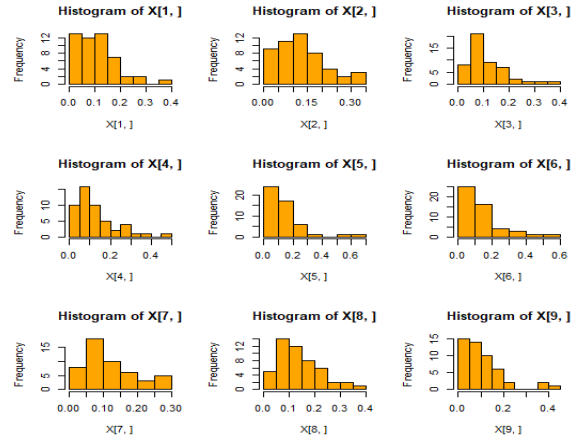


Figure 6: Histogram of simulation when $n=50$,

$\theta=15$

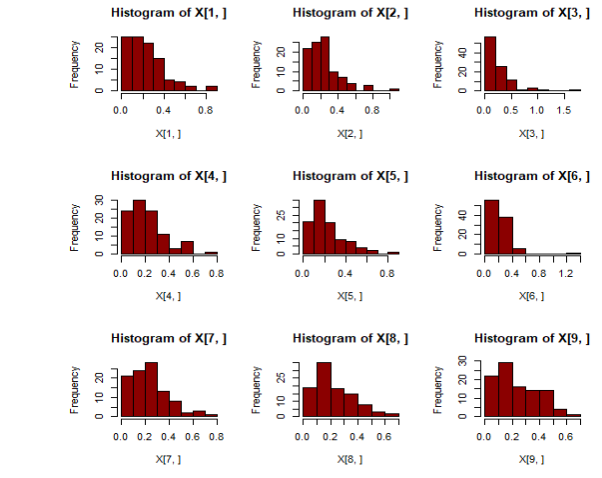


Figure 8: Histogram of simulation when $n=100$, $\theta=9$

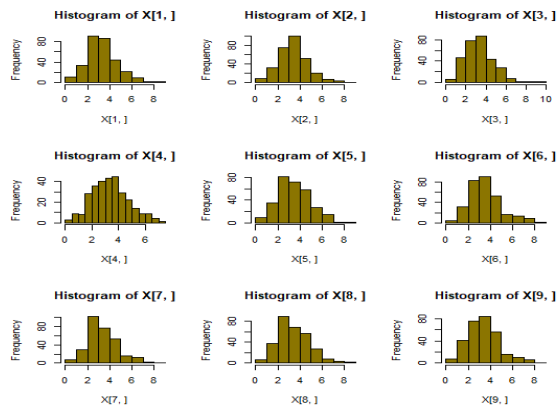


Figure 10: Histogram of simulation when $n=300$, $\theta=2$

12. APPLICATION

In this section, we use and analyse the two real-life data sets to show that the Length-biased Prakaamy distribution fits better than the Prakaamy distribution, Exponential distribution, Lindley distribution. The following two data sets are provided below as:

Data set 1: The following data represent 40 patients suffering from blood cancer (leukemia) from one of Ministry of Health Hospitals in Saudi Arabia (see Abouammah *et al.*). The ordered lifetimes (in years) are given below

0.315, 0.496, 0.616, 1.145, 1.208, 1.263, 1.414, 2.025, 2.036, 2.162, 2.211, 2.37, 2.532, 2.693, 2.805, 2.91, 2.912, 3.192, 3.263, 3.348, 3.348, 3.427, 3.499, 3.534, 3.767, 3.751, 3.858, 3.986, 4.049, 4.244, 4.323, 4.381, 4.392, 4.397, 4.647, 4.753, 4.929, 4.973, 5.074, 5.381

Data set 2: Relief in minute’s analgesic data of 20 patients has been reported by Gross and Clark in (1975).

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2,
1.4, 3.0, 1.7, 2.3, 1.6, 2.0

Estimates of the unknown parameters are carried out in R software along with calculation of model comparison criterion values like AIC, AICC, BIC and $-2\log L$ values. In order to compare the two models, the AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion) are used. The better distribution corresponds to lesser AIC, AICC and BIC values. The generic formulas for calculation of AIC, AICC and BIC are

$$AIC = 2k - 2\log L; \quad AICC = AIC + \frac{2k(k+1)}{n-k-1} \quad \text{and} \quad BIC = k \log n - 2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and $-\text{Log}L$ is the maximized value of the log-likelihood function under the considered model. Table 2 shows the parameter estimation and standard error values. Table 3 shows the performance of the distributions. Figure 11 shows Fitting density curves of data set 1 based on data of patients suffering from blood cancer (leukemia) and Figure 12 shows Fitting density curves of data set 2 based on relief in minutes of analgesic patients.

Table 2: Shows values of ML estimates, and corresponding standard errors

Data Set	Distribution	Parameter	MLE	Standard Error
1	LBP	θ	2.1361574	0.1188292
	Prakaamy	θ	1.72885043	0.09691529
	Exponential	θ	0.31839887	0.05034279
	Lindley	θ	0.52692133	0.06074766
2	LBP	θ	3.011022	0.207976
	Prakaamy	θ	2.273508	0.161589
	Exponential	θ	0.5263164	0.1176875
	Lindley	θ	0.8161188	0.1360929

Table 3: Shows values of $2\log L$, AIC, BIC, and AICC

Data Set	Distribution	$-2\log L$	AIC	BIC	AICC
1	LBP	139.7248	141.7248	143.4137	141.8332632
	Prakaamy	140.6615	142.6615	144.3504	142.7667632
	Exponential	171.5563	173.5563	175.2452	173.6615632
	Lindley	160.5012	162.5012	164.19	162.6064632
2	LBP	49.50585	51.50585	52.50158	51.72807222
	Prakaamy	61.43961	63.43961	64.43534	63.66183222
	Exponential	65.67416	67.67416	68.66989	67.89638222
	Lindley	60.4991	62.4991	63.49483	62.72132222

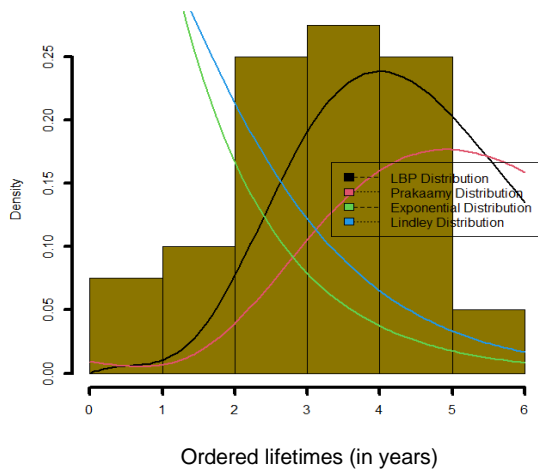


Figure 11: Fitting density curves of data set-1 of patients suffering from blood cancer (leukemia)

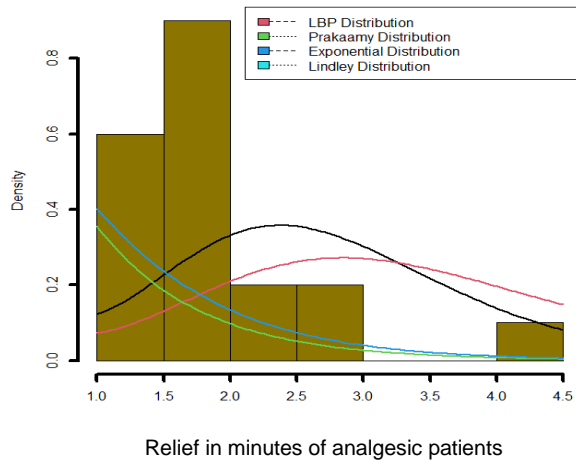


Figure 12: Fitting density curves of data set-2 based on relief in minutes of analgesic patients

From Table 3, it has been observed that the Length-biased Prakaamy distribution has smaller AIC, AICC, -LogL and BIC values as compared to the Prakaamy distribution, Exponential distribution, and Lindley distribution, which clearly indicates that Length-biased Prakaamy distribution fits better than Prakaamy distribution, Exponential distribution, and Lindley distribution. Hence we can conclude that the Length-biased Prakaamy distribution leads to a better fit than the above distributions.

CONCLUSION

In the present study we have studied a Length-biased Prakaamy distribution as a new generalization of Prakaamy distribution. The new distribution is generated by using the weighting technique and taking the one parameter Prakaamy distribution as the base distribution. Some mathematical properties along with reliability measures are discussed. The hazard rate function and reliability behaviour of the Length-biased Prakaamy distribution exhibits that subject distribution can be used as a lifetime model. Finally, real life data has been analysed for comparisons purpose, and it has been analysed that Length-biased Prakaamy distribution shows better performance than Prakaamy, Exponential, and Lindley distributions.

In the present study we have studied a Length-biased Prakaamy distribution as a new generalization of the Prakaamy distribution. This new distribution is created by employing a weighting technique and using the one-parameter Prakaamy distribution as the foundation. The research delves into various mathematical properties and discusses reliability measures associated with this distribution. The findings suggest that the Length-biased Prakaamy distribution's hazard rate function and reliability behaviour make it suitable as a lifetime model.

Moreover, the study includes an analysis of real-life data for comparison purposes. It indicates that the Length-biased Prakaamy distribution more beneficial than other distributions, such as Prakaamy, Exponential, and Lindley distributions.

This suggests that the Length-biased Prakaamy distribution may offer a more accurate and useful model for analysing and predicting the behaviour of certain real-world data compared to the other distributions mentioned. The term "better performance" indicates that it might have a better fit or provide more accurate predictions for the specific dataset analysed in the study. However, it's important to consider that the suitability of a distribution depends on the specific characteristics of the data being analysed, and the choice of distribution should be based on the data's underlying properties and assumptions.

REFERENCES

- [1] Abouammoh, A.M., Ahmed, R. and Khalique, A. (2000). On new renewal better than used classes of life distribution. *Statistics and Probability Letters*, 48, 189-194.
- [2] Afaq, A., Ahmad, S. P., and Ahmed, A. (2016). Length-Biased weighted Lomax distribution: Statistical properties and applications, *Pak.j.Stat.Oper.res.*, vol.xii no. 2, pp 245-255.
- [3] Cox D. R. (1969). Some sampling problems in technology, In *New Development in Survey Sampling*, Johnson, N. L. and Smith, H., Jr. (eds.) New York Wiley- Interscience, 506-527.
- [4] Dey, S., Ali, S., & Park, C. (2015). Weighted exponential distribution: properties and different methods of estimation. *Journal of Statistical Computation and Simulation*, 85(18), 3641-3661
- [5] Fisher, R. A. (1934), The effects of methods of ascertainment upon the estimation of frequencies. *Annals of Eugenics*, 6, 13-25.
- [6] Gross, A.J. and Clark, V.A., (1975), *Survival Distributions: Reliability Applications in the Biometrical Sciences*, John Wiley, New York.
- [7] Khan, M. N., Saeed, A., & Alzaatreh, A. (2018). Weighted Modified Weibull distribution. *Journal of Testing and Evaluation*, 47(5), 20170370.
- [8] Kilany, N. M. (2016). Weighted Lomax distribution. *SpringerPlus*, 5(1). doi:10.1186/s40064-016-3489-2
- [9] Mathew, J. & Chesneau, C. (2020), Marshall–olkin length-biased Maxwell distribution and its applications, *Mathematical and Computational Applications*, 25(4), 65. doi:10.3390/mca25040065.
- [10] Modi, K. and Gill, V., (2015), Length-biased Weighted Maxwell Distribution, *Pak.j.stat.oper.res.* Vol.XI No.4, pp465-472.
- [11] Mudasir, S. & Ahmad, S. P., (2018), Characterization and estimation of length biased nakagami distribution, *Pak.j.stat.oper.res.* Vol.698 XIV No.3 , pp697-715
- [12] Rather, A. A., and Subramanian, C., (2019), The Length-Biased Erlang–Truncated Exponential Distribution with Life Time Data, *Journal of Information and Computational Science*, Volume 9 Issue 8, pp 340-355.
- [13] Rather, A. A., Subramanian, C., Shafi, S., Malik, K. A., Ahmad, P. J., Para, B. A. and Jan, T. R. (2018), A new Size Biased Distribution with applications in Engineering and Medical Science, *International Journal of Scientific Research in Mathematical and Statistical Sciences*, Vol.5, Issue.4, pp.75-85.
- [14] Reyad, M. H., Hashish, M. A., Othman, A. S. & Allam, A. S. (2017), The length-biased weighted frechet distribution: properties and estimation, *international journal of statistics and applied mathematics*, 3(1), pp 189-200.
- [15] Shenbagaraja, R., Rather, A. A. and Subramanian, C., (2019), On Some Aspects of Length Biased Technique with Real Life Data, *Science, Technology and Development*, Vol VIII, issue IX, pp 326-335.
- [16] Subramanian, C and Rather, A. A. (2018).Weighted exponentiated mukherjee-islam distribution, *International journal of management, technology and engineering*, vol. 8, issue xi, pp. 1328-1339.
- [17] Zelen, M. (1974). Problems in cell kinetic and the early detection of disease, in *Reliability and Biometry*, F. Proschan & R. J. Sering, eds, SIAM, Philadelphia, 701-706.

DOI: <https://doi.org/10.15379/ijmst.v10i1.2861>

This is an open access article licensed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/3.0/>), which permits unrestricted, non-commercial use, distribution and reproduction in any medium, provided the work is properly cited.