

Inventory System with Time and Price Dependent Demand Ratio for Decreasing Items

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Abstracts: An inventory model for decaying items with time and price dependent demand for horizon is developed. In this model deterioration rate is taken as quadratic as well as shortages are not permitted. Cost minimization technique is used to get the approximate expressions for total cost and other parameters. we developed a model under consider demand as price and time dependent rate. Special cases of model are discussed for different values of change in constant of proportional and inflation rate.

Keywords: Decaying Items, Time and Price-Dependent Demand Rates, Inventory Model, Deterioration Rate, Cost Minimization Technique.

1. INTRODUCTION

This document includes an EOQ model discussing time and the time-varying quadratic degradation rate for a single item, which is dependent on both time and price. Prior research has focused mostly on time-dependent demand, including linear, quadratic, and exponential models. The primary drawback of the demand rate's linear time dependency is that it presupposes a uniform change in the demand rate per unit of time. In the case of any marketable commodity, this seldom occurs. A more accurate description of time-varying market needs appears to be quadratic time-dependence of demand.

However, an exponential rate of change in demand is extremely high, and any commodity's demand fluctuation on the actual market cannot be so high. In the opinion of the current writers, the function of selling price and time may best express demand. In addition, a quadratic rate of decline has been included. This research may therefore be directly applied to a variety of real-world circumstances.

Products constantly degrade in real-world settings. In recent years, a number of academics have investigated how items deteriorate. The first piece was created by Ghare and Schrader. [1] who created the model for a product whose inventory degrades rapidly. Since that time, deteriorating researchers have gained a lot of notoriety. Convert and Philip [3] loosened up the constant rate of degradation supposition. [3] who created the EOQ model for goods with a two-parameter Weibull distribution for the degradation pattern. Philip further generalized this concept. [4] By using a Weibull distribution with three parameters, The EOQ model for products with Weibull distribution degradation, scarcity, and trended demand by Chakrabarty, Giri, and Chaudhuri is a development of Philip's model. [14]. Maiti, Mondal, and Bhunia [21] are regarded it as an inventory system that improves things for demand rates that depend on pricing. Wee and Law [25] proposed a time-discounting-based integrated production-inventory model for improving and decaying products. Lo, Wee, and Huang later [26] extension of the integrated production-inventory model with flawed production processes and deteriorating Weibull distribution under inflation as opposed to Tadikamalla [5] assuming that the gamma distribution controls degradation. It is quite difficult to do research on time-dependent genuine market-oriented demand. Meal and Silver [2] For the broad scenario of deterministic, time-varying demand patterns, a rough solution approach was constructed. Patel and Dave [6] established a demand-proportional inventory model for degrading goods Hariga expanded on this work. [9] Jalan, Giri, and Chaudhari [11], Jalan and Chaudhari [15], Giri and Chaudhari [12] Lin, Tan, and Lee [16], Wang [18] and Ghosh and Chaudhari [22], etc

Inventory shortages are a relatively common occurrence in practical circumstances. Roy Chowdhary and Chaudhari [7] created a finite replenishment rate inventory model. Wee [8] created a manufacturing lot size model for partially back-ordered products that are degrading. Wee [10] is taken into account as an inventory model for deteriorating goods with shortages and exponentially declining demand rates over a limited planning horizon. Alyan and Hariga [13] created a novel heuristic that is computationally and economically more effective than Wee's method. [10]. Researchers made later contributions in this direction. Papa Christos and Skouri [17], Wang [19], Goyal and Giri [20], Zhou, Lau, and Yang [23], and Song and Chan [24]. Sicilia et al [27] presented a model with power law demand rate and uniform replenishment. Heydari et al. [28] presented a model taking price sensitive rate. Xu et al. [29] present a model for permissible product considering stock dependence demand. Wang et al. [30] consider time dependent demand for deteriorating item in their model.

In this paper, we developed a model under consider demand as price and time dependent rate.

2. ASSUMPTIONS AND NOTATIONS

This model is made in accordance with the following assumptions and notations –

- (i) "C" is the constant price per unit of purchasing of time.
- (ii) "k" is the cost of ordering per cycle.
- (iii) "h" is the cost of keeping inventory per unit of time.
- (iv) "p" the selling price per unit of the product
- (v) $d(p, t) = d(p)e^{\mu t}$, $0 \leq \mu < 1$, is the price time-dependent demand rate having a

negative derivative w.r.t. to 'p' in its entire domain. Here μ is the inflation rate.

(vi) $\lambda(t) = a + bt + ct^2$, $0 \leq a < 1$, $0 \leq b < 1$, $0 \leq c < 1$ represents the time-proportional rate of deterioration. where a, b, c are the constants of proportional due to pollutions, light and temperature conditions of the stock house respectively.

(vii) The variable 'T' represents the duration of each cycle.

(viii) Shortages are not permitted

3. THE PROBLEM AND ITS PROPOSED SOLUTION

Let $I(t)$ be the current inventory level at time $t \geq 0$. Then the differential equation below describes the current state of $I(t)$ at any time 't'.

$$\begin{aligned} \frac{dI(t)}{dt} &= -\lambda(t)I(t) - d(p, t) \\ &= -(a + bt + ct^2)I(t) - d(p)e^{\mu t}, 0 \leq t \leq T \end{aligned} \tag{1}$$

Under the condition

$$I(T) = 0 \tag{2}$$

Here in the demand function $d(p, t)$, $d(p)$ is the function of p only.

Now, the solution of equation (1) is given by

$$I(t) = -d(p) \left[t + \frac{1}{2}(\mu + a)t^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right] e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} + I(0)e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} \quad (3)$$

Where $I(0)$ represents the initial stock at the time of any decay. ignoring the higher-level powers of a , b , c , and μ higher level then one degree also multiplying any two, three or four of them. Let $Z(t)$ represents the loss in stock due to decay in the time interval $[0, t]$, [See Appendix A]

$$Z(t) = I(t) \left[e^{(at + bt^2 + \frac{1}{3}ct^3)} - 1 \right] + d(p) \left(\frac{1}{2}at^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right) \quad (4)$$

Using equation (2) in equation (4), we have

$$Z(t) = d(p) \left(\frac{1}{2}aT^2 + \frac{1}{6}bT^3 + \frac{1}{12}cT^4 \right)$$

Also, the total demand in one run is

$$\begin{aligned} &= \int_0^T d(p, t) dt = d(p) \int_0^T e^{\mu t} dt \\ &= d(p) \int_0^T (1 + \mu t) dt = d(p) \left(T + \frac{1}{2}\mu T^2 \right) \end{aligned}$$

[Neglecting the power of μ higher than one]

Now, since demand or decay are the only causes of loss in the inventory system. Therefore, the ordered quantity in each cycle is

$$\begin{aligned} Q_T &= Z(T) + d(p) \left(T + \frac{1}{2}\mu T^2 \right) \\ &= d(p) \left[T + \frac{1}{2}(\mu + a)T^2 + \frac{1}{6}bT^3 + \frac{1}{12}cT^4 \right] \end{aligned} \quad (5)$$

Since $I(0) = Q_T$

Therefore, from equation (3) we get

$$I(t) = d(p) \left[\begin{aligned} &(T - t) + \frac{1}{2}(\mu + a)(T^2 - t^2) + \frac{1}{6}b(T^3 - t^3) \\ &+ \frac{1}{12}c(T^4 - t^4) \end{aligned} \right] e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} \quad (6)$$

$0 \leq t \leq T$

Clearly it satisfies boundary condition i.e. $I(T) = 0$

Now the total cost for each cycle is [See Appendix B]

$$\begin{aligned}\bar{C}(T, p) &= K + \hat{C}Q_T + h \int_0^T I(t)dt \\ &= K + \hat{C}d(p) \left[T + \frac{1}{2}(\mu + a)T^2 + \frac{1}{6}bT^3 + \frac{1}{12}cT^4 \right] \\ &\quad + hd(p) \left[\frac{1}{2}T^2 + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^3 + \frac{1}{12}bT^4 + \frac{1}{20}cT^5 \right]\end{aligned}\quad (7)$$

As a result, the average cost of system is

$$\begin{aligned}C(T, p) &= \frac{K}{T} + \hat{C}d(p) \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] \\ &\quad + hd(p) \left[\frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right]\end{aligned}\quad (8)$$

Additionally, the order rate per unit of time is

$$\frac{Q_T}{T} = d(p) \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right]\quad (9)$$

Now, the order rate's behaviour to variations in demand in a, b, c and μ is determined by

$$\frac{\partial}{\partial a} \left(\frac{Q_T}{T} \right) = \frac{1}{2}Td(p) > 0$$

$$\frac{\partial}{\partial b} \left(\frac{Q_T}{T} \right) = \frac{1}{6}T^2d(p) > 0$$

$$\frac{\partial}{\partial c} \left(\frac{Q_T}{T} \right) = \frac{1}{12}T^3d(p) > 0$$

And

$$\frac{\partial}{\partial \mu} \left(\frac{Q_T}{T} \right) = \frac{1}{2}Td(p) > 0$$

Also, with respect to price "p" is

$$\frac{\partial}{\partial p} \left(\frac{Q_T}{T} \right) = d'(p) \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] \leq 0,$$

$$\text{Since } d'(p) \leq 0 \quad (10)$$

Therefore, the optimal ordering rate increases with increases in a, b, c, and μ and decrease with increase in p.

Now, the profit rate is

$\Pi(T, p)$ = Average selling cost – Average system cost

$$\begin{aligned}&= pd(p) \left(1 + \frac{1}{2}\mu T \right) - \frac{K}{T} - \hat{C}d(p) \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] \\ &\quad - hd(p) \left[\frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right]\end{aligned}\quad (11)$$

Our current task is to determine the values of T and p that maximize $\Pi(T, p)$

We now take

$$d(p) = \alpha p^{-\beta}, \quad \alpha > 0, \quad \beta > 0 \tag{12}$$

Here α represents the scale parameter and β represents the parameter that defines the shapes of the demand curve.

Using relation (12) in the relation (8) and relation (11), we have

$$C(T, p) = \frac{K}{T} + \hat{C}\alpha p^{-\beta} \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] + h\alpha p^{-\beta} \left[\frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right] \tag{13}$$

And

$$\begin{aligned} \Pi(T, p) = & \alpha p^{1-\beta} \left(1 + \frac{1}{2}\mu T \right) - \frac{K}{T} - \hat{C}\alpha p^{-\beta} \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] \\ & - h\alpha p^{-\beta} \left[\frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right] \end{aligned} \tag{14}$$

The required condition to maximization the $\Pi(T, p)$ is

$$\frac{\partial \Pi(T, p)}{\partial T} = 0 = \frac{\partial \Pi(T, p)}{\partial p}$$

The required condition for maximization of $\Pi(T, p)$ are necessary that it should be a concave function for $T > 0, p > 0$

The concavity condition (See Appendix C) on $\Pi(T, p)$ is

$$\begin{aligned} & \left[\frac{2k}{T^3} + \hat{C}\alpha p^{-\beta} \left(\frac{1}{3}b + \frac{1}{2}cT \right) + h\alpha p^{-\beta} \left\{ \frac{2}{3}\left(\mu + \frac{1}{2}a\right) + \frac{1}{2}bT + \frac{3}{5}cT^2 \right\} \right] \\ & \left[\begin{array}{l} \alpha\beta(1-\beta)p^{-\beta-1} + \hat{C}\alpha\beta(\beta+1)p^{-\beta-2} \\ \left\{ 1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right\} + h\alpha\beta(\beta+1)p^{-\beta-2} \\ \left\{ \frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right\} \end{array} \right] \\ & > \left[\begin{array}{l} \frac{1}{2}\alpha(1-\beta)\mu p^{-\beta} + \hat{C}\alpha\beta p^{-\beta-1} \left\{ \frac{1}{2}(\mu + a) + \frac{1}{3}bT + \frac{1}{4}cT^2 \right\} \\ + h\alpha\beta p^{-\beta-1} \left\{ \frac{1}{2} + \frac{3}{2}\left(\mu + \frac{1}{2}a\right)T + \frac{1}{4}bT^2 + \frac{1}{5}cT^3 \right\} \end{array} \right]^2 \end{aligned} \tag{15}$$

Now, $\frac{\partial \Pi(T,p)}{\partial T} = 0$, gives

$$\frac{1}{2}\mu\alpha p^{1-\beta} + \frac{K}{T^2} - C^{\wedge}\alpha p^{-\beta} \left[\frac{1}{2}(\mu + a) + \frac{1}{3}bT + \frac{1}{4}cT^2 \right] - h\alpha p^{-\beta} \left[\frac{1}{2} + \frac{2}{3}\left(\mu + \frac{1}{2}a\right)T + \frac{1}{4}bT^2 + \frac{1}{5}cT^3 \right] = 0 \tag{16}$$

And

$\frac{\partial \Pi(T,p)}{\partial p} = 0$, gives

$$\alpha(1 - \beta)p^{-\beta} \left(1 + \frac{1}{2}\mu T \right) + C^{\wedge}\alpha\beta p^{-\beta-1} \left[1 + \frac{1}{2}(\mu + a)T + \frac{1}{6}bT^2 + \frac{1}{12}cT^3 \right] + h\alpha\beta p^{-\beta-1} \left[\frac{1}{2}T + \frac{1}{3}\left(\mu + \frac{1}{2}a\right)T^2 + \frac{1}{12}bT^3 + \frac{1}{20}cT^4 \right] = 0 \tag{17}$$

Solving Equation Number (16) and (17) simultaneously and find the values of T and p for which function $\Pi(T, p)$ will be maximum.

4. SOME SPECIAL CASES

Case I:

$$\text{If } \mu, a, b \neq 0 \quad \text{and} \quad c = 0$$

i.e., There is necessary temperature condition but pollutions and light problem present.

Case II:

$$\text{If } \mu, a, c \neq 0 \quad \text{and} \quad b = 0$$

i.e., There is no light problem.

Case III:

$$\text{If } \mu, b, c \neq 0 \quad \text{and} \quad a = 0$$

i.e., There is no pollution.

Case IV:

$$\text{If } a, b, c \neq 0 \quad \text{and} \quad \mu = 0$$

i.e., Demand is the function of p only.

Case V:

$$\text{If } a, \mu \neq 0 \quad \text{and} \quad b = c = 0$$

i.e., There are no light and temperature problems.

Case VI:

$$\text{If } b, \mu \neq 0 \quad \text{and} \quad a = c = 0$$

i.e., There are no pollution and temperature problems.

Case VII:

$$\text{If } c, \mu \neq 0 \quad \text{and} \quad a = b = 0$$

i.e., There are no pollution and light problems.

Case VIII: If $a = b = c=0$ but $\mu \neq 0$

i.e., There is no decay.

CONCLUSION

This paper presents the development of an inventory model and analysed for items with time and price dependent demand factors. In this model deterioration rate is taken as quadratic. Cost minimization technique is used to get the approximation expressions for total cost and other parameters. Special cases of model are also discussed for different values of a, b, c and μ .

Appendix A:

The current inventory level at any given time $t \in [0, t]$, where there is decay, from equation (3), we have

$$I(t) = -d(p) \left[t + \frac{1}{2}(\mu + a)t^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right] e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} + I(0)e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} \tag{A1}$$

So that

$$\bar{I}(t) = I(t)e^{(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} + d(p) \left[t + \frac{1}{2}(\mu + a)t^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right] \tag{A2}$$

Let $I(t)$ represents the current inventory level at any given time $t \in [0, t]$, at no decay, we have,

$$\bar{I}(t) = \lim_{a,b,c \rightarrow 0} I(t) = I(0) - d(p) \left(t + \frac{1}{2}\mu t^2 \right) \tag{A3}$$

The stock loss due to decay during a specified time interval $[0, t]$

Then

$$Z(t) = \bar{I}(t) - I(t) = I(0) - d(p) \left(t + \frac{1}{2}\mu t^2 \right) + d(p) \left[t + \frac{1}{2}(\mu + a)t^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right] e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} - I(0)e^{-(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} \tag{A4}$$

From equation (A2) and equation (A4), we have

$$Z(t) = I(t) \left[e^{(at + \frac{1}{2}bt^2 + \frac{1}{3}ct^3)} - 1 \right] + d(p) \left(\frac{1}{2}at^2 + \frac{1}{6}bt^3 + \frac{1}{12}ct^4 \right) \tag{A5}$$

Appendix B:

The total cost of each cycle is

$$\begin{aligned} \bar{C}(T, p) &= \text{Ordering cost} + \text{Purchase cost} + \text{holding cost} \\ &= K + \hat{C}d(p) \left[T + \frac{1}{2}(\mu + a)T^2 + \frac{1}{6}bT^3 + \frac{1}{12}cT^4 \right] \\ &\quad + hd(p) \left[\frac{1}{2}T^2 + \frac{1}{3} \left(\mu + \frac{1}{2}a \right) T^3 + \frac{1}{12}bT^4 + \frac{1}{20}cT^5 \right] \end{aligned}$$

Appendix C:

The profit function of Concavity condition $\Pi(T, p)$, is required here to ensure the existence of a unique point of maximum for $\Pi(T, p)$,

We have

$$\begin{aligned} \Pi(T, p) &= \alpha p^{1-\beta} \left(1 + \frac{1}{2} \mu T \right) - \frac{K}{T} - \hat{C} \alpha p^{-\beta} \left[1 + \frac{1}{2} (\mu + a) T + \frac{1}{6} b T^2 + \frac{1}{12} c T^3 \right] \\ &\quad - h \alpha p^{-\beta} \left[\frac{1}{2} T + \frac{1}{3} \left(\mu + \frac{1}{2} a \right) T^2 + \frac{1}{12} b T^3 + \frac{1}{20} c T^4 \right] \\ \frac{\partial \Pi(T, p)}{\partial T} &= \frac{1}{2} \mu \alpha p^{1-\beta} + \frac{K}{T^2} - \hat{C} \alpha p^{-\beta} \left[\frac{1}{2} (\mu + a) + \frac{1}{3} b T + \frac{1}{4} c T^2 \right] \\ &\quad - h \alpha p^{-\beta} \left[\frac{1}{2} + \frac{2}{3} \left(\mu + \frac{1}{2} a \right) + \frac{1}{4} b T^2 + \frac{1}{5} c T^3 \right] \\ \frac{\partial \Pi(T, p)}{\partial p} &= \alpha (1 - \beta) p^{-\beta} \left(1 + \frac{1}{2} \mu T \right) + \hat{C} \alpha \beta p^{-\beta-1} \left[1 + \frac{1}{2} (\mu + a) T + \frac{1}{6} b T^2 + \frac{1}{12} c T^3 \right] \\ &\quad + h \alpha \beta p^{-\beta-1} \left[\frac{1}{2} T + \frac{1}{3} \left(\mu + \frac{1}{2} a \right) T^2 + \frac{1}{12} b T^3 + \frac{1}{20} c T^4 \right] \\ \frac{\partial^2 \Pi(T, p)}{\partial T^2} &= -\frac{2K}{T^3} - \hat{C} \alpha p^{-\beta} \left(\frac{1}{3} b + \frac{1}{2} c T \right) - h \alpha \beta \left[\frac{2}{3} \left(\mu + \frac{1}{2} a \right) + \frac{1}{2} b T + \frac{3}{5} c T^2 \right] \\ \frac{\partial^2 \Pi(T, p)}{\partial p^2} &= -\alpha \beta (1 - \beta) p^{-\beta-1} \left(1 + \frac{1}{2} \mu T \right) - \hat{C} \alpha \beta (\beta + 1) p^{-\beta-2} \left[1 + \frac{1}{2} (\mu + a) T + \frac{1}{6} b T^2 + \frac{1}{12} c T^3 \right] \\ &\quad - h \alpha \beta p^{-\beta-2} \left[\frac{1}{2} T + \frac{1}{3} \left(\mu + \frac{1}{2} a \right) T^2 + \frac{1}{12} b T^3 + \frac{1}{20} c T^4 \right] \end{aligned} \tag{C1}$$

And

$$\begin{aligned} \frac{\partial^2 \Pi(T, p)}{\partial T \partial p} &= \frac{\partial^2 \Pi(T, p)}{\partial p \partial T} \\ \frac{1}{2} \mu \alpha (1 - \beta) p^{-\beta} + \hat{C} \alpha \beta p^{-\beta-1} &\left[\frac{1}{2} (\mu + a) + \frac{1}{3} b T + \frac{1}{4} c T^2 \right] \\ + h \alpha \beta p^{-\beta-1} &\left[\frac{1}{2} + \frac{2}{3} \left(\mu + \frac{1}{2} a \right) T + \frac{1}{4} b T^2 + \frac{1}{5} c T^3 \right] \end{aligned} \tag{C3}$$

Now the function $\Pi(T, p)$ will be concave if

$$\begin{bmatrix} \frac{\partial^2 \Pi(T, p)}{\partial T^2} & \frac{\partial^2 \Pi(T, p)}{\partial T \partial p} \\ \frac{\partial^2 \Pi(T, p)}{\partial p \partial T} & \frac{\partial^2 \Pi(T, p)}{\partial p^2} \end{bmatrix} > 0 \tag{C4}$$

And

$$\frac{\partial^2 \Pi(T, p)}{\partial T^2} < 0 \tag{C5}$$

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