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Comparison of Accelerated Life Tests for Two Different Systems Generated Using the Arrhenius Model

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Abstracts: Accelerated testing consists of a variety of methods aimed at shortening the life of a product or hastening its performance degradation in a short period of time. The purpose of these tests is to quickly obtain data that, when properly modeled and analyzed, can provide the desired information about a product's lifetime or performance under normal use. Accelerated life testing will subject a product to conditions that exceed normal service parameters in order to uncover defects and potential failure modes. Since the guidelines for statistical analysis of thermal life test data in accelerated life tests (ANSI/IEEE Std. 101-1987) were enacted, they have been used to analyze the results of many experiments. However, Shim (2004) only used a small amount of data obtained from a limited environment for analysis of ANSI/IEEE Std. 101-1987 results, so simulations were conducted using previous research results to supplement the accuracy of the analysis. In this paper, two different systems or materials generated using the Arrhenius equation are compared in an accelerated life test to estimate the correct lifetime.

Keywords: Acceleration Life Test, Arrhenius Model, Different Temperatures, Lifetime, Pooled Standard Deviation, Regression Model.

1. THE ARRHENIUS EQUATION AND ITS APPLICATION

In physical chemistry, the Arrhenius equation is a formula to determine the temperature dependence of reaction rates. This equation was proposed by Svante Arrhenius in 1889 for temperature dependence of the equilibrium constant was formulated for both forward and backward reactions.

The Arrhenius equation has extensive and important applications in determining chemical reaction rates and calculating activation energies.

Arrhenius provided a physical justification and interpretation for the formula. Based on Arrhenius's law for simple chemical reaction rates, this relationship is used to describe many products that fail as a result of chemical reactions or metal-diffusion-induced decomposition. This relationship is appropriate over a specific temperature range.

According to Arrhenius's law, the rate of a simple chemical reaction depends on temperature as follows:

$$\tau = \mathrm{D} \exp\left(-E/kT\right) \qquad (1)$$

where

τ is the chemical reaction rate

D is a constant that is characteristic of the product failure mechanism and test conditions

E is the activation energy of the reaction, usually in electron-volts

k is the Boltzmann constant (8.6171×10^{-5} electron-volts per ˚C)

T is the absolute Kelvin temperature

Metal diffusion rates are described by the same equation. Therefore, the following Arrhenius lifetime relation, based on (1), can describe diffusion-induced failures in solid-state devices and certain other products made of metal.

The following relationship is based on a simple view of failures due to these chemical reactions. If a certain critical amount of chemical reacts, the product is assumed to have failed. A brief view of this is

$$\log(L) = constant + \left((E/kT)/2.303\right) \qquad (2)$$

where L is the median lifetime of metal samples.

Equation (2) is expressed in algebraic form as

$$\mathrm{Y} = \mathrm{P} + \mathrm{QX} \qquad (3)$$

where

Y is Log(L), and X is 1/T

P is a constant characteristic of the test sample, test method, and failure mode, and Q is E/(2.303R).

In Equation (3), the coefficients P and Q can be estimated from the experimental data, so we denote the sample estimated values as p and q.

Theoretically, Equation (3) is valid only when the chemical reaction is simple and the failure mode controls the insulation failure mode. Doing so is effective. Usually one response and failure mode appears over a range of temperatures, but other responses with different temperature coefficients and/or failure modes are evident at low or high temperatures. In the simple case represented by the Arrhenius model, deviations may be caused by different failure modes at different temperatures or by changes in mechanical stress that affect life with temperature, so the processing of life-temperature data for insulators is important. We can say that the application of the Arrhenius model is appropriate.

In order to compare data from different sample tests, or duplicate tests on the same sample, it is desirable to analyze them at various temperatures, in which case both the mean and the standard deviation of the log life at different temperatures can be calculated.

Let L_{ij} be the lifetime of sample *j* at temperature *i*, and let its logarithm be Y_{ij}. Then,

$$Y_{ij} = \log\left(L_{ij}\right)$$

and the average, $\bar{Y}$, of Y_{ij} at any temperature i is

$$\bar{Y}_i = \frac{1}{n_i}\sum_{j=1}^{k} Y_{ij} \qquad (4)$$

where n_i is the number of samples at temperature i.

The sample variance of the log-time-to-failure at any temperature is

$$s_i^2 = \frac{1}{n_i-1}\sum_{j=1}^{k}\left(Y_{ij} - \bar{Y}_i\right)^2 \qquad (5)$$

2. Estimating the relationship between lifetime and temperature

The assumptions for applying the Arrhenius model to the analysis of thermal life test data, and estimating the change in life according to temperature, are as follows. First, the log life relationship to the reciprocal Kelvin temperature is linear over the temperature range of interest. Second, the lifetime of the sample is statistically independent. Third, the sample is randomly selected from the population of interest. Fourth, any variation in log life has a normal distribution with the same standard deviation at all temperatures of interest.

Based on the above assumptions, convert the Celsius test temperature of each sample to its inverse Kelvin temperature as follows:

$$\mathrm{X} = 1/(T + 273) \quad (6)$$

Since the failure life, L, of each sample is converted to $\mathrm{Y} = \log(\mathrm{L})$, the estimated values of the population parameters in Equation (3) are as follows:

$$\mathrm{p} = \bar{Y} - q\bar{X} \quad (7)$$

$$\mathrm{q} = r\{s_y/s_x\} \quad (8)$$

where *r* is the sample correlation coefficient of variables x and y, and s_x and s_y are the standard deviations of variables *x* and *y*, respectively.

Letting $\mathrm{m}(T_c)$ be the sample estimate of the population average log life for a selected temperature T_c , it can be calculated using the following equation:

$$\mathrm{m}(T_c) = p + q[1/(T_c + 273)] \quad (9)$$

The antilog of $\mathrm{m}(T_c)$ is an estimate of the median lifetime per unit of time at temperature T_c.

Calculate the estimated value for the population standard deviation, σ, of the log life as follows:

$$\mathrm{s} = \sqrt{\textstyle\sum_{i=1}^{k}(Y_i - (p + qX_i))^2/(n-2)} \quad (10)$$

Calculate the variance of selected temperature T_c from:

$$\mathrm{V}(T_c) = (X_c - \textstyle\sum_{i=1}^{k} X_i/n)^2/\{\sum_{i=1}^{k} X_i^2 - (\sum_{i=1}^{k} X_i)^2/n\} \quad (11)$$

where $X_c = 1/(T_c + 273)$.

The upper confidence limit, $m_u(T_c)$, and the lower confidence limit, $m_l(T_c)$, for average log lifetime $\mathrm{m}(T_c)$ can be calculated as follows using equations (10) and (11):

$$m_u(T_c) = (p + qX_c) + t_{n-2}s\sqrt{(1/n) + Var(T_c)} \quad (12)$$

$$m_l(T_c) = (p + qX_c) - t_{n-2}s\sqrt{(1/n) + Var(T_c)} \quad (13)$$

3. Comparison of two average log lifetimes at different temperatures

Thermal lifetime testing can be used to compare the lifetimes of two different systems or materials. After obtaining the data for both substances, the straight line obtained by transforming the Arrhenius equation can be used. Using this straight line, it is possible to observe the presence or absence of comparable characteristics, such as whether one insulation system is noticeably better than another. Observed differences may be due to chance changes. To assess whether there are acceptable differences, you need to show how to compare the differences between two sets of data, similar to the procedure for comparing two averages.

Comparisons between straight lines can usually be made at more than one temperature, and comparisons at temperatures within a range of tests may be useful in some cases.

Let us define the following:

$\mathrm{m}(T_c) =$ the average log lifetime estimated at temperature T_c , $c = 1{,}2$

$s_c =$ the standard deviation of lifetime estimated at temperature T_c , $c = 1{,}2$

$n_1, n_2 =$ the number of samples from each group used in the experiment

Standard deviation s1 and s2, and variance in the results obtained at experimental temperatures 1 and 2 to be compared, can be obtained. First, the pooled standard deviation for the two insulators is calculated as follows:

$$s_p = \sqrt{\{(n_1 - 2)s_1^2 + (n_2 - 2)s_2^2\}/(n_1 + n_2 - 4)} \qquad (14)$$

The average log lifetime, $\mathrm{m}(T_c), c = 1{,}2$, for each straight line at the selected temperature is estimated using Equation (9). Using the values obtained above, the statistic required to obtain the confidence interval is calculated with the following formula, which follows the distribution with $n_1 + n_2 - 4$ degrees of freedom:

$$\mathrm{t} = \{m(T_1) - m(T_2)\} \,/\, \left\{s_p\sqrt{((n_1 + n_2)/n_1 n_2) + Var_1(T_1) + V_{ar2}(T_2)}\right\} \qquad (15)$$

As another expression of Equation (15), the confidence interval for the difference between average log lifetimes at experimental temperature t_c, can be calculated as follows:

$$m_1(T_1) - m_2(T_2) \pm t\left\{s_p\sqrt{((n_1 + n_2)/n_1 n_2) + Var_1(T_1) + Var_2(T_2)}\right\} \qquad (16)$$

If this interval contains 0, the two average log lifetimes do not have a statistically significant difference, and if the interval does not contain 0, the average log lifetimes have a statistically significant difference. In most cases, note that it is easy to treat them as the same, even though the average log lifetimes of the two insulators obtained in actual experiments are often different.

CONCLUSION

The case where the effect of stress on lifetime follows a linear model has been studied a lot. However, the linear model cannot be used when causing a chemical reaction such as external temperature affecting the lifetime of the subject (such as an insulator). In this case, the Arrhenius model can be used. The Arrhenius model was used to represent a section where there is a difference in lifespan when two chemical stresses exist. Using this method, it is helpful to estimate the lifespan of the subject material according to the movement of nearby stress.

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