An Integer Programming Model for a Nurse Scheduling Problem

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Abstracts: This paper addresses a nurse scheduling problem that is present in a hospital. In order to accommodate the rising cost of sustaining and maintaining the hospital, there is a demand to analyse the manpower scheduling problem by not compromising the needs of the patients. Therefore, this study aims to apply a mathematical method to find the optimal number of nurses to be assigned for duties each day and shift in a hospital. An integer linear programming model is developed to represent the nurse scheduling problem. The mathematical model is solved by using the QM for windows software and the results provides an optimal solution to the nurse scheduling problem.

Keywords: nurse, scheduling, linear programming.

1. INTRODUCTION

Healthcare scheduling problem provides a huge challenge due to high constraints and limitations to medical service and resource requirements such as medical supplies, equipment, doctors and nurses [1]. Nurses scheduling plays a significant role and has been of great concern to many hospital administrators in many healthcare institutions. Most of the hospitals still practices manual methods to schedule nurses or staff even though this method is labour intensive and time consuming besides providing poor work schedules in most cases [2,3]. The inefficiency nurse staffing and irregular work shifts too may lead to overworking manpower and job dissatisfaction which directly influences patients' dissatisfaction. In the real world problem, some of the factors that can influence the complexity of the nurse scheduling problem are unpredicted absenteeism, emergency leave applications and variation in patients' demands [4,5,6].

Heuristic approach for a multi-objective nurse scheduling problem was developed in [6] which produces a flexible and ease of implementation on spreadsheets. Furthermore, they solved the model by using a commercial software (CPLEX) which provides same results. They also indicated that manual scheduling does not provide the optimal results and application of commercial software are too expensive to be implemented. A mathematical programming model to minimize nurses total idle waiting time was developed to represent a nurses labour shifts [7]. The developed model is solved to optimality by using LINGO software for a numerical example. Many methods have been used by researches to solve the nurse scheduling problem such as mathematical programming techniques, heuristics and metaheuristics and simulation [8]. A hybrid integer programming and variable neighbourhood search algorithm had been developed to address the nurse scheduling problem according to the actual demand of the hospital with some constraints taken into consideration such as nursing time required by patients and effective nurse working hours [9]. A new nurse rostering problem was investigated in [10] which takes into account of additional constraints to account for the multi-stage setting of the scheduling problem. Their proposal of the extension made to the integer programming formulations showed improvement upon the results of their basic models proposed earlier in [11]. During the Covid-19 pandemic, the management of nurse scheduling faced new and complex challenges and a hike in operation costs of the healthcare industry [12]. Their paper proposed a combination of optimization and decision algorithm methods which enables the medical staff to provide their best services to the patients in the hospitals. The trend for future studies will be more focused on scientific methods to solve nurse scheduling problems.

In this paper, a mathematical model based on integer programming is developed to address the nurse scheduling problem in a hospital. The mathematical model is solved by using the QM for windows software. The paper is organized as follows. First, the specific problem statement and the mathematical model of the nurse scheduling problem is provided in the section 'Methodology'. It is followed by the section 'Results and Discussion'.

Finally, the conclusion and future research is provided in the ‘Conclusion’ section.

2. METHODOLOGY

2.1. Problem Statement

A nursing requirements in a large hospital which consists of many medical specialists’ department is presented here. A one week planning horizon for nurses are considered. The daily requirements of nurses are different for each shift and days according to the needs and various operational hours for every department. Here, all nurses are assumed to have equal qualifications and skills to be assigned in any shifts or departments. Nurses are required to work for 2 shifts continuously (8 hours) and are required to report for duty at the beginning of each period during working hours. Each day consists of 3 shifts. Shift 1 starts at 8 a.m. and ends at 12 p.m. Shift 2 begins at 12 p.m. and ends at 4 p.m. Shift 3 begins at 4 p.m. and ends at 8 p.m. Night shifts nurses after 8 p.m. for warded patients are not considered here. Furthermore, the nurses work for 5 days consecutively in a week and are given 2 days off. Table **1** gives the minimum number of nurses required for a week schedule. The objective is to determine the optimal (minimal) number of nurses to be employed and to have a sufficient number of nurses available for each shift.

Table 1. Minimum number of Nurses Required on each shift and day.

Shift	Minimum number of Nurses Required						
	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	80	110	120	75	55	50	40
2	90	100	115	70	60	45	37
3	100	95	110	90	55	40	30

2.2. Mathematical Model

Here, two mathematical models are presented. Each model consists of decision variables, objective function and constraints. The first mathematical model aims to minimize the number of nurses per day when the nurses work 5 days consecutively. The second model presents the mathematical model to determine the number of nurses for each shift if they work for 2 shifts consecutively.

2.2.1 Model 1

Let x_i be the nurses who starts work on day *i* where *i* = 1, 2,..7 . For example, x_1 represents nurses who starts work on Monday. Table **2** provides the allocation of the nurses on working days and the days off.

Table 2. Nurses who starts work on a day and their off day.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Required
Mon	/(start)	Off	Off	/	/	/	/	270
Tues	/	/(start)	Off	Off	/	/	/	305
Wed	/	/	/(start)	Off	Off	/	/	345
Thurs	/	/	/	/(start)	Off	Off	/	235
Fri	/	/	/	/	/(start)	Off	Off	170
Sat	Off	/	/	/	/	/(start)	Off	135
Sun	Off	Off	/	/	/	/	/(start)	107

The integer mathematical model is developed by setting up the objective function and the appropriate constraints.

Objective function

Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$...(Eq 0)

Subject to Constraints

$$x_1 + x_4 + x_5 + x_6 + x_7 \geq 270 \text{...(Eq 1)}$$
$$x_1 + x_2 + x_5 + x_6 + x_7 \geq 305 \text{...(Eq 2)}$$
$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 345 \text{...(Eq 3)}$$
$$x_1 + x_2 + x_3 + x_4 + x_7 \geq 235 \text{...(Eq 4)}$$
$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 170 \text{...(Eq 5)}$$
$$x_2 + x_3 + x_4 + x_5 + x_6 \geq 135 \text{...(Eq 6)}$$
$$x_3 + x_4 + x_5 + x_6 + x_7 \geq 107 \text{...(Eq 7)}$$
$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \text{...(Eq 8)}$ and all variables are integers.

Equation (0) provides the objective function *Z* which aims to minimize the number of nurses per day when the nurses work 5 days consecutively. Equations 1 (Eq 1) to 7 (Eq 7) ensures that sufficient nurses are working for Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday respectively by not violating the minimum required nurses. The last constraint gives the non-negativity constraint and all variables are integers.

2.2.2. Model 2

Here, the model is developed to determine the number of nurses for each shift if they work for 2 shifts consecutively. By referring to Table **3**, there are 21 decision variables introduced to represent the starting shift on each working day. For example, w_1 represents the number of nurses who starts work on Monday on shift 1, w_2 represents the number of nurses who starts work on Monday on shift 2 and w_{21} represents the number of nurses who starts work on Sunday on shift 3.

Table 3. Number of nurses who starts work on a shift on each day.

Shift/Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	w_1	w_4	w_7	w_{10}	w_{13}	w_{16}	w_{19}
2	w_2	w_5	w_8	w_{11}	w_{14}	w_{17}	w_{20}
3	w_3	w_6	w_9	w_{12}	w_{15}	w_{18}	w_{21}

Let $c_1, c_2, c_3, c_4, c_5, c_6$ and c_7 be the optimal number of nurses required each day for Monday**,** Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday. The integer mathematical model is developed by setting up the objective function and the appropriate constraints**.**

Objective Function

Minimize $Y = w_1 + w_2 + w_3 + ... + w_{19} + w_{20} + w_{21} \text{...(Eq 0)}$

Subject to Constraints

$$w_1 + w_{21} \geq 80 \text{...(Eq 1)}$$
$$w_1 + w_2 \geq 90 \text{...(Eq 2)}$$
$$w_2 + w_3 \geq 100 \text{...(Eq 3)}$$
$$w_3 + w_4 \geq 110 \text{...(Eq 4)}$$
$$w_4 + w_5 \geq 100 \text{...(Eq 5)}$$
$$w_5 + w_6 \geq 95 \text{...(Eq 6)}$$
$$w_6 + w_7 \geq 120 \text{...(Eq 7)}$$
$$w_7 + w_8 \geq 115 \text{...(Eq 8)}$$
$$w_8 + w_9 \geq 110 \text{...(Eq 9)}$$

$$w_9 + w_{10} \geq 75 \ldots (\text{Eq } 10)$$
$$w_{10} + w_{11} \geq 70 \ldots (\text{Eq } 11)$$
$$w_{11} + w_{12} \geq 90 \ldots (\text{Eq } 12)$$
$$w_{12} + w_{13} \geq 55 \ldots (\text{Eq } 13)$$
$$w_{13} + w_{14} \geq 60 \ldots (\text{Eq } 14)$$
$$w_{14} + w_{15} \geq 55 \ldots (\text{Eq } 15)$$
$$w_{15} + w_{16} \geq 50 \ldots (\text{Eq } 16)$$
$$w_{16} + w_{17} \geq 45 \ldots (\text{Eq } 17)$$
$$w_{17} + w_{18} \geq 40 \ldots (\text{Eq } 18)$$
$$w_{18} + w_{19} \geq 40 \ldots (\text{Eq } 19)$$
$$w_{19} + w_{20} \geq 37 \ldots (\text{Eq } 20)$$
$$w_{20} + w_{21} \geq 30 \ldots (\text{Eq } 21)$$
$$w_1 + w_2 + w_3 \leq c_1 \ldots (\text{Eq } 22)$$
$$w_4 + w_5 + w_6 \leq c_2 \ldots (\text{Eq } 23)$$
$$w_7 + w_8 + w_9 \leq c_3 \ldots (\text{Eq } 24)$$
$$w_{10} + w_{11} + w_{12} \leq c_4 \ldots (\text{Eq } 25)$$
$$w_{13} + w_{14} + w_{15} \leq c_5 \ldots (\text{Eq } 26)$$
$$w_{16} + w_{17} + w_{18} \leq c_6 \ldots (\text{Eq } 27)$$
$$w_{19} + w_{20} + w_{21} \leq c_7 \ldots (\text{Eq } 28)$$

$w_1, w_2 \ldots w_{20}, w_{21} \geq 0 \ldots (\text{Eq } 29)$ and all variables are integers.

Equation (0) provides the objective function *z* which aims to minimize the number of nurses per shift for each day. Equations 1 (Eq 1) to 21 (Eq 21) ensures that sufficient nurses are working for each shift for each day if the nurses are working consecutively for 2 shifts. For example, the first constraint (Eq1) ensures that the number of nurses who work in shift 1 on Monday need to be at least 80. Since the nurses work 2 shifts continuously, we have to take into consideration nurses who starts work on Monday shift 1 (w_1) and the nurses who starts work on shift 3 on Sunday (w_{21}). Equations 22 (Eq 22) to 28 (Eq 28) ensures that total number of nurses do not exceed the optimal number of nurses for each day. The last constraint gives the non-negativity constraint and all variables are integers.

3. RESULTS AND DISCUSSIONS

Both the integer programming models were solved using the QM for windows (V5) to find the optimal solutions. Figure **1** provides the output produced based on model 1.

Integer & Mixed Integer Programming Results

(untitled) Solution

Variable	Type	Value
X1	Integer	210
X2	Integer	28
X3	Integer	0
X4	Integer	0
X5	Integer	0
X6	Integer	107
X7	Integer	0
Solution value		345

Figure 1. Output based on Model 1.

Based on Figure **1**, the optimal requirement of nurses per day can be calculated as shown in Table **4.**

Table 4. Optimal number of nurses per day.

Optimal requirement of nurses per day (based on results in Figure 1)	
Monday	210+107=317 (c_1)
Tuesday	210+28+107=345 (c_2)
Wednesday	210+28+107=345 (c_3)
Thursday	210+28+0=238 (c_4)
Friday	210+28+0=238 (c_5)
Saturday	28+107=135 (c_6)
Sunday	107 (c_7)

Hence, these values of $c_1, c_2, c_3, c_4, c_5, c_6$ and c_7 will be used as parameters in Model 2 to generate solutions. Figure **2** provides the output for Model 2. The figure provides the variables in the notations of ‘X” which refers to “w” in Table **3**. For example, X1 refers to w_1, X2 refers to w_2, X21 refers to w_{21} etc.

Integer & Mixed Integer Programming Results

(untitled) Solution

Variable	Type	Value
X1	Integer	217
X2	Integer	100
X3	Integer	0
X4	Integer	250
X5	Integer	95
X6	Integer	0
X7	Integer	235
X8	Integer	110
X9	Integer	0
X10	Integer	148
X11	Integer	90
X12	Integer	0
X13	Integer	183
X14	Integer	55
X15	Integer	0
X16	Integer	95
X17	Integer	40
X18	Integer	0
X19	Integer	77
X20	Integer	0
X21	Integer	30
Solution value		1725

Figure 2. Output based on Model 2.

The optimal number of nurses for each shift and each day is provided in Figure **2**. The highest number of nurses start work on Tuesday on shift 1 which is a total of 250 nurses. None of the nurses start their work shifts on Shift 3 on Monday, Tuesday, Wednesday and Thursday and shift 2 on Sunday. Most of the nurses prefers to start their work in shift 1. There are total of 1725 nurses required for the 1 week of schedule.

4. CONCLUSION

A nurse scheduling problem has been analyzed and studied in this paper. An optimization method based on integer programming model has been developed to find solutions to an illustrated example which resembles a hospital needs on nursing staff for each shift and day. The developed mathematical model is able to produce optimal solutions in mere seconds by using the QM for windows software which can be installed at no extra cost. The advantages of this method enables the scheduling of nurses to be developed quickly without compromising the quality of the output besides saving cost compared to the frequent usage of the manual distribution of nurse scheduling. Future research can be focused on studying real life data from hospitals which would be more realistic and beneficial to both the healthcare institution and to the scholars related to this area. Metaheuristics such as tabu search or simulated annealing could be an alternative solution method when handling a more complex and larger problems in nurse scheduling. Having said this, the integer programming model developed in this paper could be used in other workforce scheduling models such as transportation industry and other service industries.

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