Method of Green's Functions for the Problem of Sound Diffraction on Elastic Shell of Non-Analytical Form

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Abstract: The real scatterers have a non–analytical form and, therefore, the variable separation method (Fourier series method) for calculation of the reflected sound field cannot be applied to them. This article presents the method of Green's functions and methods of the dynamic theory of elasticity for the solution of the problem of sound diffraction on elastic shell of a non–analytical surface. Furthermore, this work includes the detailed analysis of the solution of this problem and calculation of the angular characteristics of the sound scattering by non-analytical scatterers.

Keywords: Diffraction, Method of Green's functions, Non–analytical surface, Boundary conditions, Scatterer.

INTRODUCTION

There are quite a number of methods to solve problems of reflection and scattering of sound bodies of a non–analytical surface. The most well-known and frequently used of them: methods of finite and boundary elements, method Kupradze, the method of the T – matrix, method of geometrical theory of diffraction, the method of integral equations, the method of Green's functions, etc. [1-25]. In this paper we use the method of Green's functions [22-25], which was developed for the solution of diffraction problems on the phone with mixed boundary conditions, and first applied in this study of the sound scattering by non-analytical scatterers.

THE SOLUTION OF THE THREE-DIMENSIONAL PROBLEM OF DIFFRACTION ON AN ELASTIC CYLINDRICAL SHELL USING THE DEBYE POTENTIALS

We consider bodies, the surface of which cannot be applied to the category of coordinate systems with shared variables in the scalar Helmholtz equation, as non-analytical bodies. Let's consider this non-analytical scatterer in the form of a finite circular cylinder, bounded on the sides by spheroidal caps (Figure **1**).

Sound pressure, scattered by this body, can be found by one of the numerical methods for solving the diffraction problems, among which the method of Green's functions is sufficiently convenient. This method is based on the use of mathematical formulation of the principle of Helmholtz-Huygens (Kirchhoff integral). The algorithm of calculation requires knowledge of the amplitude-phase distribution of the sound pressure and the normal component of oscillatory velocity on some closed integration surface of S that in this case consists of the lateral surface of the cylinder S_2 and the surfaces of spheroidal caps S_1 and S_3 (Figure **1**):

$$p_S(P) = \frac{-1}{4\pi}\int_S [p_S(Q)\frac{\partial}{\partial n}G(P,Q) - \frac{\partial p_S(Q)}{\partial n}G(P,Q)]dS, \quad (1)$$

where $p_s(P)$ - the sound pressure scattered by the body, P – the observation point having spherical coordinates; r, Θ, φ – the point on the surface S; $p_s(Q)$ - the sound pressure in the point Q; $G(P,Q)$ - the Green's function of the free space satisfying the inhomogeneous Helmholtz equation.

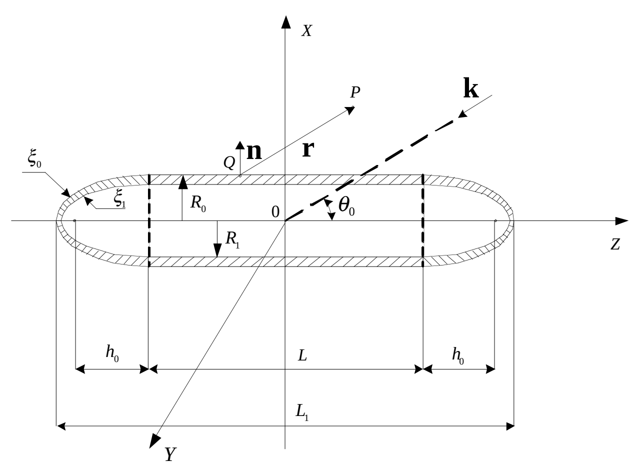

Figure 1: Elastic non-analytical shell, compiled from finite cylindrical shell with spheroidal shells on the ends**.**

In the formula (1) the Green's function is selected as a point source potential:

$$G(P,Q) = \frac{e^{ikR}}{R} \quad (2)$$

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where $k = \dfrac{2\pi}{\lambda}$ the wave number, λ - the length of the sound wave in the liquid medium, *R* – the distance between the points P and Q.

First, consider problem of sound diffraction in infinite elastic hollow cylindrical shell [13, 26, 27]. Geometry of a problem is introduced on Figure **2.**

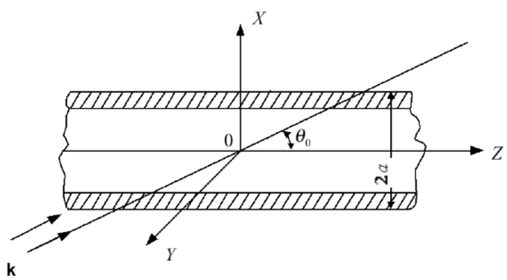

Figure 2: Infinite cylindrical shell.

The scalar potential of sound wave $\Phi_i(r,\varphi,z)$ with unit wave vector $\vec{k}$, which directed to axis *z* at angle θ, can be expand by natural functions of the scalar Helmholtz equation in circular cylindrical coordinate system:

$$\Phi_i(r,\varphi,z) = e^{i\gamma z}\sum_{m=0}^{\infty}\varepsilon_m(-i)^m J_m(k_\gamma r)\cos m\varphi, \tag{3}$$

where

$$\gamma = k\cos\theta\,;\; k_\gamma = k\sin\theta\,;\; \varepsilon_m = \begin{cases} 1 \text{ for } m = 0; \\ 2 \text{ for } m \neq 0; \end{cases}$$

Let's transform the expression for vector function **A**, which was presented at [26], using the operator *rot,* for compliance with the conditions div **A** = 0:

$$\mathbf{A} = rot(\chi\,\mathbf{k}) + \mathrm{rot}\,\mathrm{rot}\,(\psi\,\mathbf{k}), \tag{4}$$

where **k** – unit vector on axis *z*; *χ* and *ψ* – scalar potentials, satisfying the Helmholtz equation:

$$\left.\begin{aligned} \Delta\chi + k_2^2\chi = 0 \\ \Delta\psi + k_2^2\psi = 0 \end{aligned}\right\} \tag{5}$$

Components of vector function **A** in compliance with (4) have the following form:

$$\left.\begin{aligned} A_r &= \frac{1}{r}\frac{\partial\chi}{\partial\varphi} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial r\partial z}; \\ A_\varphi &= -\frac{\partial\chi}{\partial r} + \frac{1}{r}\frac{\partial^2\psi}{\partial\varphi\partial z}; \\ A_z &= -\frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2}, \end{aligned}\right\} \tag{6}$$

Components of displacement vector ***U*** will be:

$$\left.\begin{aligned} U_r &= \frac{\partial\Phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial r\partial\varphi} - \frac{1}{r}\frac{\partial^3\psi}{\partial r^2\partial\varphi} - \frac{1}{r^3}\frac{\partial^3\psi}{\partial\varphi^3} + \\ &\quad + \frac{\partial^2\chi}{\partial r\partial z} - \frac{1}{r}\frac{\partial^3\psi}{\partial\varphi\partial z^2}; \\ U_\varphi &= \frac{1}{r}\frac{\partial\Phi}{\partial\varphi} + \frac{1}{r}\frac{\partial^2\chi}{\partial\varphi\partial z} + \frac{1}{r^2}\frac{\partial^3\psi}{\partial r\partial z^2} + \frac{1}{r^2}\frac{\partial\psi}{\partial r} - \\ &\quad -\frac{1}{r}\frac{\partial^2\psi}{\partial r^2} - \frac{\partial^3\psi}{\partial r^3} + \frac{2}{r^3}\frac{\partial^2\psi}{\partial\varphi^2} - \frac{1}{r^2}\frac{\partial^3\psi}{\partial r\partial\varphi^2}; \\ U_z &= \frac{\partial\Phi}{\partial z} - \frac{1}{r}\frac{\partial\chi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi\partial z} - \frac{\partial^2\chi}{\partial r^2} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi\partial z} + \\ &\quad + \frac{1}{r}\frac{\partial^3\psi}{\partial\varphi\partial z\partial r} - \frac{1}{r^2}\frac{\partial^2\chi}{\partial\varphi^2} - \frac{1}{r^3}\frac{\partial^3\psi}{\partial r\partial z\partial\varphi}. \end{aligned}\right\} \tag{7}$$

Potentials Φ, χ, ψ, Φ_s are displayed on by natural functions of the scalar Helmholtz equation in circular cylindrical coordinate system:

$$\left.\begin{aligned} \Phi &= e^{i\gamma z}\sum_{m=0}^{\infty}\left[A_m J_m(h'r) + B_m N_m(h'r)\right]\cos m\varphi; \\ \chi &= e^{i\gamma z}\sum_{m=0}^{\infty}\left[C_m J_m(æ'r) + D_m N_m(æ'r)\right]\cos m\varphi; \\ \psi &= e^{i\gamma z}\sum_{m=1}^{\infty}\left[E_m J_m(æ'r) + F_m N_m(æ'r)\right]\sin m\varphi; \\ \Phi_s &= e^{i\gamma z}\sum_{m=0}^{\infty} G_m H_m^{(1)}(k_\gamma r)\cos m\varphi, \end{aligned}\right\} \tag{8}$$

where $h' = \left(k_1^2 - k^2\right)^{1/2}$; $æ' = \left(k_2^2 - k^2\right)^{1/2}$; $A_m, B_m, C_m, D_m, E_m, F_m, G_m$ - unknown coefficients, which are determined from the boundary conditions:

(1) The normal component of a displacement vector is continuous on the boundary of the liquid -elastic shell (*r* = *a*):

$$\begin{aligned} &\frac{\partial\Phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial r\partial\phi} - \frac{1}{r}\frac{\partial^3\psi}{\partial r^2\partial\phi} - \frac{1}{r^3}\frac{\partial^3\psi}{\partial\phi^3} + \frac{\partial^2\chi}{\partial r\partial z} - \\ &\quad -\frac{1}{r}\frac{\partial^3\psi}{\partial\phi\partial z^2} = \frac{\partial}{\partial r}(\Phi_i + \Phi_s)\Big|_{r=a} \end{aligned} \tag{9}$$

(2) The sound pressure in a liquid p_Σ is equal to the normal strain at an external boundary of elastic shell:

$$\begin{aligned} &(\lambda + 2\mu)\frac{\partial U_r}{\partial r} + \lambda\left(r^{-1}\frac{\partial U_\varphi}{\partial\varphi} + r^{-1}U_r + \frac{\partial U_z}{\partial z}\right) = \\ &= -\rho_0\omega^2(\Phi_i + \Phi_s)\big|_{r=a}, \end{aligned} \tag{10}$$

where ρ_0 - liquid density,

(3) The normal strain in a shell at an internal boundary of is equal to zero:

$$(\lambda+2\mu)\frac{\partial U_r}{\partial r}+\lambda\left(r^{-1}\frac{\partial U_\varphi}{\partial\varphi}+r^{-1}U_r+\frac{\partial U_z}{\partial z}\right)=0\Big|_{r=b} \quad (11)$$

(4) The tangent strains at the shell's boundaries are equal to zero:

$$\left.\begin{aligned}&\frac{\partial U_\varphi}{\partial r}+r^{-1}\frac{\partial U_r}{\partial\varphi}-r^{-1}U_\varphi=0\Big|_{\substack{r=a;\\ r=b}};\\ &\frac{\partial U_r}{\partial z}+\frac{\partial U_z}{\partial r}=0\Big|_{\substack{r=a\\ r=b}}.\end{aligned}\right\} \quad (12)$$

By substituting expressions (3), (8) at (7), and then at boundary conductions (9) – (12) results in the heterogeneous system of the seven equations to define the unknown coefficients of potential expansions.

As the trigonometrical functions *cos(m*φ) and *sin(m*φ*)* are opthogonal, an infinite system breaks out into seven equation with fixed index *m*: $A_m, B_m, C_m, D_m, E_m, F_m, G_m$ for finding the seven combinations of heterogeneous system.

A product G_m for a potential of a scattered wave Φ_s is calculated by Cramer's rule on a basis of a ratio of the two determinants of the seventh-order:

$$G_m=\Delta'/\Delta\,, \quad (13)$$

where Δ' - minor of the system, a Δ – system's determinant, which equals [27].

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & 0\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0\\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & 0\\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 0 \end{vmatrix}$$

$$\Delta'=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & b_{17}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & b_{27}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & 0\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0\\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0\\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & 0\\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 0 \end{vmatrix}$$

Expression of the elements of the determinants is given in the Appendix to this article.

The relationship between scattered pressure p_s and scalar potential of displacement of scattered wave Φ_s is determined by well known expression [13, 24]:

$$p_s=-\rho\omega^2\Phi_s \quad (14)$$

THE SOLUTION OF THREE-DIMENSIONAL PROBLEM OF DIFFRACTION ON ELASTIC SPHEROIDAL AND SPHERICAL SHELLS WITH THE USE OF DEBYE'S TYPE POTENTIALS

Consider the problem of diffraction of plane wave on prolate elastic spheroidal shell [13, 24, 27-28].

When studying elastic scatterers of spheroidal form irradiated of harmonic wave, are the basic equation Lame and the Helmholtz equation for scalar *Φ* and vector **A** potentials.

The solution scheme for axial-symmetric problem of sound diffraction on elastic sheroidal body (prolate or oblate) is similar to the problem of diffraction for the cylinder and sphere.

In this case, the vector potential is also one non-zero component $A = A_\varphi$. But in this case, the unknown coefficients of expansions are not in closed form, and are determined from the infinite system of equations by the method of truncation. Three-dimensional problem of diffraction of elastic spheroidal scatterer solved using the Debye's potentials *U* and *V*, through which it is expressed vector function **A** [13, 24]:

$$\mathbf{A}=rot\,rot\,(\boldsymbol{R}U)+i\,k_2\,rot\,(\boldsymbol{R}V), \quad (15)$$

where $\boldsymbol{R}$ – radius-vector of observation point, k_2 – wave number of flexural wave.

The efficiency of such conception becomes obvious, when it is considered, that potentials *U* and *V* are submitted to scalar Helmholtz equation.

First, it is convenient to write components of vector **A** in spherical coordinate system, expressing them through *U, V,* $\boldsymbol{R}$*,* and then by the formulas vector analysis go to spheroidal components.

Express spherical components of the vector function **A** through Debye's potentials [13, 24, 27-28].

$$A_{\theta_1} = \left[h_0\left(\xi^2 - 1 + \eta^2\right)\right]^{-1}\left[(\partial\xi/\partial\theta_1)(\partial\xi/\partial R)\left(\partial^2 B/\partial\xi^2\right) + \right.$$
$$+ (\partial\xi/\partial\theta_1)(\partial\eta/\partial R)\left(\partial^2 B/\partial\xi\,\partial\eta\right) +$$
$$+ (\partial\xi/\partial R)(\partial\eta/\partial\theta_1)\left(\partial^2 B/\partial\xi\,\partial\eta\right) +$$
$$+ (\partial\eta/\partial R)(\partial\eta/\partial\theta_1)\left(\partial^2 B/\partial\eta^2\right) +$$
$$+ (\partial B/\partial\xi)\left(\partial^2\xi/\partial R\,\partial\theta_1\right) +$$
$$\left. + (\partial B/\partial\eta)\left(\partial^2\eta/\partial R\,\partial\theta_1\right)\right] + ik_2\left(\sin\theta_1\right)^{-1}\left(\partial V/\partial\varphi\right); \tag{16}$$

$$A_R = (\partial\xi/\partial R)^2\left(\partial^2 B/\partial\xi^2\right) + 2(\partial\xi/\partial R)(\partial\eta/\partial R)\left(\partial^2 B/\partial\eta\,\partial\xi\right) +$$
$$+ (\partial\eta/\partial R)^2\left(\partial^2 B/\partial\eta^2\right) + \left(\partial^2\xi/\partial R^2\right)(\partial B/\partial\xi) +$$
$$+ \left(\partial^2\eta/\partial R^2\right)(\partial B/\partial\eta) + k_2^2 B; \tag{17}$$

$$A_\varphi = \left[\left(\xi^2 - 1 + \eta^2\right)^{1/2}\sin\theta_1\, h_0\right]^{-1}\left[(\partial\xi/\partial R)\left(\partial^2 B/\partial\xi\,\partial\varphi\right) + \right.$$
$$\left. + (\partial\eta/\partial R)\left(\partial^2 B/\partial\eta\,\partial\varphi\right)\right] - ik_2\left[(\partial\xi/\partial\theta_1)(\partial V/\partial\xi) + \right.$$
$$\left. + (\partial\eta/\partial\theta_1)(\partial V/\partial\eta)\right], \tag{18}$$

where $B = h_0\left(\xi^2 - 1 + \eta^2\right)^{1/2} U$.

Spheroidal components of **A** will be [13, 24, 27-28].

$$A_\xi = A_R(h_0/h_\xi)\xi(\xi^2 - 1 + \eta^2)^{1/2} +$$
$$+ A_{\theta_1}(h_0/h_\xi)(\xi^2 - 1 + \eta^2)^{1/2}(\partial\theta_1/\partial\xi);$$
$$A_\eta = A_R(h_0/h_\eta)\eta(\xi^2 - 1 + \eta^2)^{1/2} + \tag{19}$$
$$+ A_{\theta_1}(h_0/h_\eta)(\xi^2 - 1 + \eta^2)^{1/2}(\partial\theta_1/\partial\eta);$$

$A_\varphi \equiv A_\varphi$.

As scatterer consider the elastic isotropic spheroidal shell (Figure **3**). The plane wave potential, the scattered waves potential, the scalar potential of a shell , the Debye's potentials *U* and *V* are arranged in a series of spheroidal wave functions:

$$\Phi_0 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} i^{-n}\varepsilon_m \bar{S}_{m,n}(C_1,\eta_0)\bar{S}_{m,n}(C_1,\eta)R_{m,n}^{(1)}(C_1,\xi)\cos m\varphi; \tag{20}$$

$$\Phi_1 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} B_{m,n}\bar{S}_{m,n}(C_1,\eta)R_{m,n}^{(3)}(C_1,\xi)\cos m\varphi; \tag{21}$$

$$\Phi_2 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} \bar{S}_{m,n}(C_l,\eta)\cos m\varphi\left[C_{m,n}R_{m,n}^{(1)}(C_l,\xi) + D_{m,n}R_{m,n}^{(2)}(C_l,\xi)\right] \tag{22}$$

$$U = 2\sum_{m=1}^{\infty}\sum_{n\ge m}^{\infty} \bar{S}_{m,n}(C_t,\eta)\sin m\varphi\left[F_{m,n}R_{m,n}^{(1)}(C_t,\xi) + G_{m,n}R_{m,n}^{(2)}(C_t,\xi)\right] \tag{23}$$

$$V = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} \bar{S}_{m,n}(C_t,\eta)\cos m\varphi\left[H_{m,n}R_{m,n}^{(1)}(C_t,\xi) + I_{m,n}R_{m,n}^{(2)}(C_t,\xi)\right] \tag{24}$$

where $C_l = k_1 h_0$; $C_t = k_2 h_0$; $C_1 = kh_0$, *k* – the wave number of the sound wave in a gas that fills the shell; $B_{m,n}$, $C_{m,n}$, $D_{m,n}$, $E_{m,n}$, $F_{m,n}$, $G_{m,n}$, $H_{m,n}$, $I_{m,n}$ - unknown coefficients of expansion.

Unknown coefficients of expansions can be found from physical boundary conductions on both of surfaces (ξ_0 and ξ_1, Figure **3**):

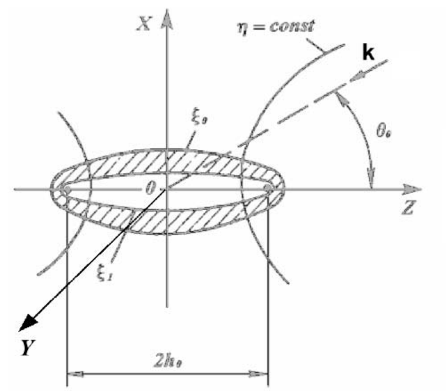

Figure 3: Elastic spheroidal shell.

(1) The normal component of a displacement vector is continuous on the external boundary $\hat{\imath}_0$; **(2)** The sound pressure in a liquid is equal to the normal strain in elastic shell; **(3)** The normal strain in a shell at an internal boundary ξ_1 of is equal to zero; **(4)** The tangent strains at the shell's boundaries (ξ_0 and ξ_1) are equal to zero.

Boundary conditions take the form accordingly [13, 24, 27-28].

$$\left(h_\xi\right)^{-1}(\partial/\partial\xi)(\Phi_0 + \Phi_1) = \left(h_\xi\right)^{-1}(\partial\Phi_2/\partial\xi) + \left(h_\eta h_\varphi\right)^{-1}\times$$
$$\times\left[(\partial/\partial\eta)\left(h_\varphi A_\varphi\right) - (\partial/\partial\varphi)\left(h_\eta A_\eta\right)\right]\Big|_{\xi=\xi_0}; \tag{25}$$

$$-\lambda_0 k^2(\Phi_0 + \Phi_1) = -\lambda_1 k_1^2\Phi_2 + 2\mu_1\left[\left(h_\xi h_\eta\right)^{-1}\times\right.$$
$$\left.\times\left(\partial h_\xi/\partial\eta\right)U_\eta + \left(h_\xi\right)^{-1}\left(\partial U_\xi/\partial\xi\right)\right]\Big|_{\xi=\xi_0}; \tag{26}$$

$$0 = -\lambda_1 k_1^2\Phi_2 + 2\mu_1\left[\left(h_\xi h_\eta\right)^{-1}\times\right.$$
$$\left.\times\left(\partial h_\xi/\partial\eta\right)U_\eta + \left(h_\xi\right)^{-1}\left(\partial U_\xi/\partial\xi\right)\right]\Big|_{\xi=\xi_1}; \tag{27}$$

$$O = \left(h_\eta/h_\xi\right)(\partial/\partial\xi)\left(U_\eta/h_\eta\right) + \left(h_\xi/h_\eta\right)(\partial/\partial\eta)\left(U_\xi/h_\xi\right)\Big|_{\substack{\xi=\xi_0\\ \xi=\xi_1}}; \tag{28}$$

$$O = \left(h_\varphi / h_\xi\right)(\partial/\partial\xi)\left(U_\varphi / h_\varphi\right) + \left(h_\xi / h_\varphi\right)(\partial/\partial\varphi)\left(U_\xi / h_\xi\right)\Big|_{\substack{\xi=\xi_0 \\ \xi=\xi_1}} . \quad (29)$$

where λ_1 и μ_1 - Lame coefficients of the shell material; λ_0 - the coefficient of volume compression of liquid.

$$U_\xi = \left(h_\xi\right)^{-1}\left(\partial\Phi_2/\partial\xi\right) + \left(h_\eta h_\varphi\right)^{-1}\left[(\partial/\partial\eta)\left(h_\varphi A_\varphi\right) - (\partial/\partial\varphi)\left(h_\eta A_\eta\right)\right];$$

$$U_\eta = \left(h_\eta\right)^{-1}\left(\partial\Phi_2/\partial\eta\right) + \left(h_\xi h_\varphi\right)^{-1}\left[(\partial/\partial\varphi)\left(h_\xi A_\xi\right) - (\partial/\partial\xi)\left(h_\varphi A_\varphi\right)\right];$$

$$U_\varphi = \left(h_\varphi\right)^{-1}\left(\partial\Phi_2/\partial\varphi\right) + \left(h_\xi h_\eta\right)^{-1}\left[(\partial/\partial\xi)\left(h_\eta A_\eta\right) - (\partial/\partial\eta)\left(h_\xi A_\xi\right)\right].$$

A substitution of the series (20) – (24) to boundary conductions (25) – (29) gives us the system of the equations to define the unknown coefficients.

As the trigonometrical functions *cos(m*φ) and *sin(m*φ) are opthogonal, an infinite system breaks out to infinite subsystems with fixed index *m*. Each of subsystems can be solved with the use of truncation method. The number of members in the ranks (20)-(24) increases with the wave size for a given potential.

Next, we consider a composite elastic shell formed by the connection of finite cylindrical shell and two hemispherical shells of the same diameter (Figure **4**).

For the application of the method of Green's functions will use the solution of the axis- symmetrical problem of diffraction of a plane acoustic wave by an elastic spherical shell under the dynamic theory of elasticity [29-31] and transform the decision on three-dimensional version.

This solution is not very different from the solution of the three-dimensional problem of diffraction on an elastic spheroidal shell [13, 24, 27-28].

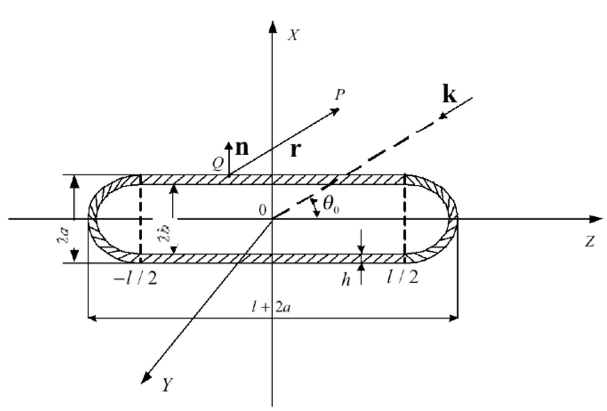

Figure 4: Elastic non-analytical shell, compiled from finite cylindrical shell with hemispheres on the ends.

The expression for the spherical components of the vector function **A**, using Debye's potentials have the following form [32-34].

$$A_R = \left(\frac{\partial^2}{\partial R^2} + k_2^2\right)(RU); \quad (30)$$

$$A_{\theta_1} = R^{-1}\frac{\partial^2}{\partial R \partial \theta_1}(RU) + ik_2(\sin\theta_1)^{-1}\frac{\partial V}{\partial\varphi}; \quad (31)$$

$$A_\varphi = (R\sin\theta_1)^{-1}\frac{\partial^2}{\partial R\partial\varphi}(RU) - ik_2\frac{\partial V}{\partial\theta_1}. \quad (32)$$

Then the solution of this problem coincides with the solution of the problem of diffraction of elastic spheroidal shell, with the wave functions of the spheroidal coordinates should be replaced with the functions of spherical coordinates [25].

For the model presented in Figure **1** were calculated modules of angular characteristic of scattering $|D(\theta)|$ at $\theta = \theta_0 = 90^0$ in range of wave sizes $kR_0 = 0,053 - 0,581$. The model had the following parameters: $L_1 = 200,51$ m; $L = 100,0$ m; $h_0 = 50,0$ m; $R_0 = 5,04$ m; $R_1 = 5,01$ m; $\xi_0 = 1,005075$; $\xi_1 = 1,005$. Under that conditions $|D(90^0)|$ changes in the range 0,49 – 18,46.

In the Figure **5** and Figure **6** the modules of angular characteristics of scattering (in the plane XOY, $\theta_0 = 90^0$) of elastic non-analytical scatterer in the form of a cylindrical shell with hemispheres on the ends (Figure **4**), for $ka = 0,523$ (Figure **5**) and for $ka = 0,941$ (Figure **6**), is presents.

The results of these calculations are very close to the characteristics of sound scattering of elastic infinite entire cylinder is given in [35].

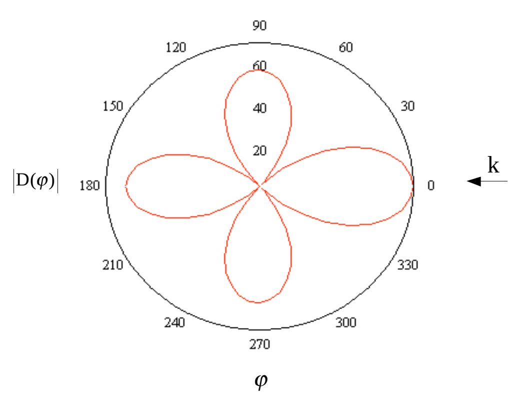

Figure 5: Module of angular characteristic of scattering $|D(\varphi)|$ for $ka = 0,523, \theta_0 = 90^0$.

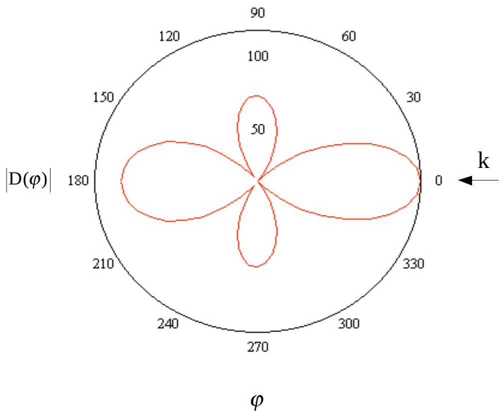

Figure 6: Module of angular characteristic of scattering $\left|D(\varphi)\right|$ for $ka = 0,941,\ \theta_0 = 90^0$.

APPENDIX

$$a_{11} = J'_m(h'a);$$

$$a_{12} = N'_m(h'a);$$

$$a_{13} = ikJ'_m(\text{æ}'a);$$

$$a_{14} = ikN'_m(\text{æ}'a);$$

$$a_{15} = a^{-1}mJ_m(\text{æ}'a)\left(a^{-2}m^2 + k^2\right) - a^{-2}mJ'_m(\text{æ}'a) - -a^{-1}mJ''_m(\text{æ}'a);$$

$$a_{16} = a^{-1}mN_m(\text{æ}'a)\left(a^{-2}m^2 + k^2\right) - a^{-2}mN'_m(\text{æ}'a) - -a^{-1}mN''_m(\text{æ}'a);$$

$$a_{17} = -H_m^{(1)\copyright}\left(k_\gamma a\right);$$

$$a_{21} = (\lambda + 2\mu)J''_m(h'a) + \lambda\left[a^{-1}J'_m(h'a) - -\left(m^2a^{-2} + k^2\right)J_m(h'a)\right];$$

$$a_{22} = (\lambda + 2\mu)N''_m(h'a) + \lambda\left[a^{-1}N'_m(h'a) - -\left(m^2a^{-2} + k^2\right)N_m(h'a)\right];$$

$$a_{23} = 2\mu\, ikJ''_m(\text{æ}'a);$$

$$a_{24} = 2\mu\, ikN''_m(\text{æ}'a);$$

$$a_{25} = (\lambda + 2\mu)a^{-1}\left\{ma^{-1}J_m(\text{æ}'a)\left(a^{-1}m^2 - k^2 - 3a^{-2}m^2\right) + +mJ'_m(\text{æ}'a)\left(2a^{-2} + k^2\right) - mJ'''_m(\text{æ}'a)\right\} + +\lambda a^{-1}\left\{a^{-1}mJ_m(\text{æ}'a)\left(k^2 - a^{-2}m^2\right) + +mJ'_m(\text{æ}'a)\left(a^{-2}m^2 - k^2\right) - 2a^{-1}mJ''_m(\text{æ}'a) - mJ'''_m(\text{æ}'a)\right\};$$

CONCLUSIONS

Using the method of Green's functions, of the Debye's potentials and "Debye's type" potentials, the solution of the problem of diffraction on an elastic composite shell of non-analytical form was obtained. The solution is supplemented by calculating angular characteristic of scattering for different wave sizes.

ACKNOWLEGMENTS

The work was supposed as part of research under State Contract no P242 of April 21.2010, within the Federal Target Program "Human Capital in Science and Education for Innovative Russia, 2009 – 2013".

$$a_{26} = (\lambda + 2\mu)\, a^{-1}\Big\{ m a^{-1} N_m(\text{æ}' a)\big(a^{-1}m^2 - k^2 - 3a^{-2}m^2\big) +$$
$$+ m N_m'(\text{æ}' a)\big(2a^{-2} + k^2\big) - m N_m'''(\text{æ}' a)\Big\} +$$
$$+ \lambda\, a^{-1}\Big\{ a^{-1} m N_m(\text{æ}' a)\big(k^2 - a^{-2}m^2\big) +$$
$$+ m N_m'(\text{æ}' a)\big(a^{-2}m^2 - k^2\big) - 2a^{-1} m N_m''(\text{æ}' a) - m N_m'''(\text{æ}' a)\Big\};$$

$$a_{27} = \rho_0 \omega^2 H_m^{(1)}\big(k_\gamma a\big);$$

$$a_{31} = (\lambda + 2\mu) J_m''(h'b) + \lambda\,\big[b^{-1} J_m'(h'b) -$$
$$- \big(m^2 b^{-2} + k^2\big) J_m(h'b)\big];$$

$$a_{32} = (\lambda + 2\mu) N_m''(h'b) + \lambda\,\big[b^{-1} N_m'(h'b) -$$
$$- \big(m^2 b^{-2} + k^2\big) N_m(h'b)\big];$$

$$a_{33} = 2\mu\, ik J_m''(\text{æ}' b);$$

$$a_{34} = 2\mu\, ik N_m''(\text{æ}' b);$$

$$a_{35} = (\lambda + 2\mu)\, b^{-1}\Big\{ m b^{-1} J_m(\text{æ}' b)\big(b^{-1}m^2 - k^2 - 3b^{-2}m^2\big) +$$
$$+ m J_m'(\text{æ}' b)\big(2b^{-2} + k^2\big) - m J_m''(\text{æ}' b)\Big\} +$$
$$+ \lambda\, b^{-1}\Big\{ b^{-1} m J_m(\text{æ}' b)\big(k^2 - b^{-2}m^2\big) +$$
$$+ m J_m'(\text{æ}' b)\big(b^{-2}m^2 - k^2\big) - 2b^{-1} m J_m''(\text{æ}' b) - m J_m'''(\text{æ}' b)\Big\};$$

$$a_{36} = (\lambda + 2\mu)\, b^{-1}\Big\{ m b^{-1} N_m(\text{æ}' b)\big(b^{-1}m^2 - k^2 - 3b^{-2}m^2\big) +$$
$$+ m N_m'(\text{æ}' b)\big(2b^{-2} + k^2\big) - m N_m'''(\text{æ}' b)\Big\} +$$
$$+ \lambda\ b^{-1}\Big\{ b^{-1} m N_m(\text{æ}' b)\big(k^2 - b^{-2}m^2\big) +$$
$$+ m N_m'(\text{æ}' b)\big(b^{-2}m^2 - k^2\big) - 2b^{-1} m N_m''(\text{æ}' b) - m N_m'''(\text{æ}' b)\Big\};$$

$$a_{37} = 0;$$

$$a_{41} = 2ma^{-1}\big[a^{-1} J_m(h'a) - J_m'(h'a)\big];$$

$$a_{42} = 2ma^{-1}\big[a^{-1} N_m(h'a) - N_m'(h'a)\big];$$

$$a_{43} = 2mika^{-1}\big[a^{-1} J_m(\text{æ}' a) - J_m'(\text{æ}' a)\big];$$

$$a_{44} = 2mika^{-1}\big[a^{-1} N_m(\text{æ}' a) - N_m'(\text{æ}' a)\big];$$

$$a_{45} = a^{-1} m^2 J_m(\text{æ}' a)\big(8a^{-3} - a^{-2}m^2 - k^2\big) +$$
$$+ a^{-3} J_m'(\text{æ}' a)\big(3k^2 - 3 - 4m^2\big) +$$
$$+ a^{-2} J_m''(\text{æ}' a)\big(3 - k^2 + 2m^2\big) +$$
$$+ a^{-1} J_m'''(\text{æ}' a) - J_m^{IV}(\text{æ}' a);$$

$$a_{46} = a^{-1}m^2 N_m(æ' a)\left(8a^{-3} - a^{-2}m^2 - k^2\right) + \\ + a^{-3}N'_m(æ' a)\left(3k^2 - 3 - 4m^2\right) + \\ + a^{-2}N''_m(æ' a)\left(3 - k^2 + 2m^2\right) + \\ + a^{-1}N'''_m(æ' a) - N^{IV}_m(æ' a);$$

$$a_{47} = 0;$$

$$a_{51} = 2mb^{-1}\left[b^{-1}J_m(h'b) - J'_m(h'b)\right];$$

$$a_{52} = 2mb^{-1}\left[b^{-1}N_m(h'b) - N'_m(h'b)\right];$$

$$a_{53} = 2mikb^{-1}\left[b^{-1}J_m(æ' b) - J'_m(æ' b)\right];$$

$$a_{54} = 2mikb^{-1}\left[b^{-1}N_m(æ' b) - N'_m(æ' b)\right];$$

$$a_{55} = b^{-1}m^2 J_m(æ' b)\left(8b^{-3} - b^{-2}m^2 - k^2\right) + \\ + b^{-3}J'_m(æ' b)\left(3k^2 - 3 - 4m^2\right) + \\ + b^{-2}J''_m(æ' b)\left(3 - k^2 + 2m^2\right) + \\ + b^{-1}J'''_m(æ' b) - J^{IV}_m(æ' b);$$

$$a_{56} = b^{-1}m^2 N_m(æ' b)\left(8b^{-3} - b^{-2}m^2 - k^2\right) + \\ + b^{-3}N'_m(æ' b)\left(3k^2 - 3 - 4m^2\right) + \\ + b^{-2}N''_m(æ' b)\left(3 - k^2 + 2m^2\right) + \\ + b^{-1}N'''_m(æ' b) - N^{IV}_m(æ' b);$$

$$a_{57} = 0;$$

$$a_{61} = 2ikJ'_m(h'a);$$

$$a_{62} = 2ikN'_m(h'a);$$

$$a_{63} = -2a^{-3}m^2 J_m(æ' a) + \\ + J'_m(æ' a)\left(a^{-2} + a^{-2}m^2 - k^2\right) - J''_m(æ' a);$$

$$a_{64} = -2a^{-3}m^2 N_m(æ' a) + \\ + N'_m(æ' a)\left(a^{-2} + a^{-2}m^2 - k^2\right) - N''_m(æ' a);$$

$$a_{65} = a^{-1}mikJ_m(æ' a)\left(a^{-2}m^2 + k^2\right) + \\ + a^{-2}mikJ'_m(æ' a)\left(3a^{-2} - 2\right) - \\ - a^{-3}mikJ''_m(æ' a);$$

$$a_{66} = a^{-1}mikN_m(\text{æ}'a)\left(a^{-2}m^2 + k^2\right) + \\ +a^{-2}mikN'_m(\text{æ}'a)\left(3a^{-2} - 2\right) - \\ -a^{-3}mikN''_m(\text{æ}'a);$$

$$a_{67} = 0;$$

$$a_{71} = 2ikJ'_m(h'b);$$

$$a_{72} = 2ikN'_m(h'b);$$

$$a_{73} = -2b^{-3}m^2J_m(\text{æ}'b) + \\ +J'_m(\text{æ}'b)\left(b^{-2} + b^{-2}m^2 - k^2\right) - J''_m(\text{æ}'b);$$

$$a_{74} = -2b^{-3}m^2N_m(\text{æ}'b) + \\ +N'_m(\text{æ}'b)\left(b^{-2} + b^{-2}m^2 - k^2\right) - N''_m(\text{æ}'b);$$

$$a_{75} = b^{-1}mikJ_m(\text{æ}'b)\left(b^{-2}m^2 + k^2\right) + \\ +b^{-2}mikJ'_m(\text{æ}'b)\left(3b^{-2} - 2\right) - \\ -b^{-3}mikJ''_m(\text{æ}'b);$$

$$a_{76} = b^{-1}mikN_m(\text{æ}'b)\left(b^{-2}m^2 + k^2\right) + \\ +b^{-2}mikN'_m(\text{æ}'b)\left(3b^{-2} - 2\right) - \\ -b^{-3}mikN''_m(\text{æ}'b);$$

$$a_{77} = 0;$$

$$b_{17} = \varepsilon_m(-i)^m J'_m\left(k_\gamma a\right);$$

$$b_{27} = -\rho_0\omega^2\varepsilon_m(-i)^m J_m\left(k_\gamma a\right).$$

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Received on 04-05-2014 Accepted on 08-05-2014 Published on 28-08-2014

http://dx.doi.org/10.15379/2408-977X.2014.01.01.2