Magnetized Anisotropic Dark Energy Bianchi Type III Cosmological Models in Brans-Dicke Theory of Gravitation

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Abstract: We investigate the spatially homogeneous Bianchi Type III cosmological models with magnetized anisotropic dark energy fluid in the scalar tensor theory of gravitation proposed by Brans-Dicke [1]. The solutions of the models are obtained by volumetric exponential expansion, power law expansion and power law relation between scalar field ϕ and scale factor '*a*'. The physical aspects of the dark energy models are discussed.

Keywords: Bianchi Type III Universe, Brans-Dicke Theory of Gravitation, Dark Energy.

1. INTRODUCTION

The standard cosmological model suggests that total energy density of the universe is dominated by two components namely dark matter and dark energy, which drives the accelerated expansion [2]. Most considered dark energy models present an accelerated expansion due to the presence of a quintessence or a phantom field. Its existence was confirmed by several high precisions observational experiments, [3-9] especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment. The WMAP shows that dark energy occupies about 73 % of the energy of our universe and dark matter about 23%. The usual baryon matter, which can be described by our known particle theory, occupies only about 4% of the total energy of the universe. Therefore, dark energy models have significant importance as far as theoretical study of the universe is concerned. It is characterized by the equation of state (EoS) $p = w\rho$, where p is a pressure of fluid, ρ is the energy density of fluid and w is a function of the cosmic time *t* only [10]. Recent cosmological observation [11] from SNe Ia data indicate that (w) is not constant and it suggests that $(-1.67 < w < -0.62)$ while the limit imposed on (w) by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics is $(-1.33 < w < -0.79)$. The simplest DE candidate is the vacuum energy $(w = -1)$, which is cosmological constant (Λ). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence $(w > -1)$, phantom energy $(w < -1)$ and quintom as evolved and have time dependent EoS parameter.

Several authors [12-15] studied dark energy models for Bianchi Type III cosmological model in the context of general relativity. Among the various modifications of general relativity (GR), the scalar-tensor theories of gravitation proposed by Brans-Dicke [1], Nordvedt [16], Lyra [17], Sen and Dunn [18] and Saez-Ballester [19]. It is a well known that, for better understanding of the early stages of evolution of the universe; the scalar–tensor theories of gravitation play a vital role. Brans-Dicke (BD) theory of gravity is a well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor. The Brans-Dicke [1] field equations are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,k}\phi^{,k}\right) - \frac{1}{\phi}\left(\phi_{;\mu\nu} - g_{\mu\nu}\phi^{;k}_{,k}\right) = \frac{8\pi T_{\mu\nu}}{\phi}, \quad (1)$$

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi\eta T}{(3+2\omega)\phi} \quad (2)$$

where ω is a dimensionless coupling constant. In this theory one extra parameter ω is used which satisfies the equation (2). The function ϕ is known as BD scalar field while *T* is the trace of the matter energy – momentum tensor. It should be note that the general relativity is recovered in the limiting case $\omega \to \infty$. Thus, we can compare our results with experimental tests for significantly large value of ω.

A detailed discussion of BD cosmology is given by Singh *et al.* [20]. The study of Bianchi type models in the context of BD theory has attracted many authors in the recent years [21]. Lorenz-Petzold [22] studied exact Bianchi type –III solutions in the presence of electromagnetic field. Kumar *et al.* [23] investigated perfect fluid solution using Bianchi type I space –time in scalar –tensor theory. Adhav *et al.* stuidied LRS Bianchi Type II cosmological model with anisotropic dark energy [24]. Katore *et al.* [25, 26] have discussed

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Bianchi Type V and Plane symmetric space-time filled with dark energy models in Brans-Dicke theory of gravitation. Bianchi Type III dark energy model in scalar tensor theory of gravitation explained by Naidu *et al.* [27]. Rao *et al.* [28] investigated LRS Bianchi Type I dark energy cosmological model in Brans-Dicke theory of gravitation. Adhav *et al.* [29] explored Bianchi type III cosmological model with negative constant deceleration parameter in Brans-Dicke theory of gravity in the presence of perfect fluid. Anisotropic dark energy Bianchi Type III cosmological models in Brans-Dicke theory of gravity is obtained by Shamir *et al.* [30]. Bianchi Type III cosmological models with anisotropic dark energy is investigated by Akarsu *et al.* [31].This motivates us to investigate Bianchi Type III cosmological model with magnetized anisotropic dark energy in Brans-Dicke theory of gravitation.

In this paper, we focuss our attention to explore the solutions of magnetized anisotropic dark energy Bianchi type III cosmological model in the context of BD theory of gravity. We find the solutions using the assumption of exponential law, power law and power law relation between scalar field and scalar factor. The paper has the following format: In section 2, the metric and field equations are described. The solutions of field equations are presented in section 3 and section 4. Sections 5 conclude the findings.

2. METRIC AND FIELD EQUATIONS

Spatially homogeneous and anisotropic cosmological models play a vital role in the study of the early stages of evolution of the universe. We consider a spatially homogeneous Bianchi type III space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2dz^2 , \tag{3}$$

where *A, B, C* are metric potentials and function of the cosmic time *t* only. We assume that the universe is filled with anisotropic fluid and that there is no electric field while the magnetic field is oriented along *x*-axis. King *et al.* [32] used the magnetized perfect fluid energy–momentum tensor to discuss the effects of magnetic field on the evolution of the universe. The simplest generalization of the EoS parameter of a perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy-momentum tensor in a consistent way with the considered metric. Hence the combined energy-momentum tensor for anisotropic fluid and magnetic field is taken in the following form

$$T^{\mu}_{\nu} = diag[T^1_1, T^2_2, T^3_3, T^4_4] , \tag{4}$$

$$T^{\mu}_{\nu} = diag[-p_x + \rho_B, -p_y - \rho_B, -p_z - \rho_B, \rho + \rho_B] , \tag{5}$$

where ρ is the energy density of the fluid while p_x, p_y and p_z are the pressures on x, y and z axes respectively, ρ_B stands for energy density of magnetic field and w_x, w_y, w_z are the directional EoS parameters of the fluid. w is the deviation free EoS parameter of the fluid.

Now parametrizing, the deviation from isotropy by setting $w_x = w$ and then introducing skewness parameter δ and γ *i.e.* the deviation from w respected on both the y and z axes. Here δ and γ are not necessarily constants and can be functions of the cosmic time *t*.

From equation (5), we have

$$T^{\mu}_{\nu} = diag[-w\rho + \rho_B, -(w+\delta)\rho - \rho_B, -(w+\gamma)\rho - \rho_B, \rho + \rho_B] \tag{6}$$

If the deviation parameters are zero, then equation (6) represents the energy-momentum tensor for the isotropic fluid and magnetic field [32]. For zero magnetic fields, equation (6) is reduced to be the energy-momentum tensor of anisotropic dark energy fluid [30-31].

In the co-moving co-ordinate system, the Brans-Dicke field equations (1) and (2) for the metric (3) with the help of equation (4) becomes

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{\phi_{44}}{\phi} = \frac{8\pi}{\phi}\left[-w\rho + \rho_B\right] \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{A_4}{A} + \frac{C_4}{C}\right) + \frac{\phi_{44}}{\phi} = \frac{8\pi}{\phi}\left[-(w+\delta)\rho - \rho_B\right] \tag{8}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{A_4}{A} + \frac{B_4}{B}\right) + \frac{\phi_{44}}{\phi} = \frac{8\pi}{\phi}\left[-(w+\gamma)\rho - \rho_B\right] \tag{9}$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{1}{A^2} - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{\phi}\left[\rho + \rho_B\right] \tag{10}$$

$$\frac{B_4}{B} - \frac{A_4}{A} = 0 . \tag{11}$$

Here the sub indices '4' in A, B, C and elsewhere denote derivatives with respect to cosmic time *t*.

The equation of motion

$$T^{ij}_{;j}=0, \tag{12}$$

is consequence of the field equations (1) and (2).

Equation (12) gives us

$$\rho_4+(1+w)\rho\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)+\rho\left(\frac{-\delta}{A^2}+\delta\frac{B_4}{B}+\gamma\frac{C_4}{C}\right)+(\rho_B)_4+2\left(\frac{-1}{A^2}+\frac{B_4}{B}+\frac{C_4}{C}\right)\rho_B=0. \tag{13}$$

Using equation (2), we get

$$\phi_{44}+\phi_4\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi(1-3w-\delta-\gamma)\rho}{(3+2\omega)\phi}. \tag{14}$$

From equation (11), without loss of generality, we obtain

$$A=B. \tag{15}$$

We substitute the value of equation (15) in equation (8) and subtract the result from (7), we obtain the skewness parameter on *y*-axis as

$$\delta=\frac{-2\rho_B}{\rho}. \tag{16}$$

Thus, the system of equations (7)-(11), (13) and (14) reduce to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{\phi_{44}}{\phi}=\frac{8\pi}{\phi}\left[-w\rho+\rho_B\right] \tag{17}$$

$$2\frac{B_{44}}{B}+\frac{B^2{}_4}{B^2}-\frac{1}{B^2}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(2\frac{B_4}{B}\right)+\frac{\phi_{44}}{\phi}=\frac{8\pi}{\phi}\left[-(w+\gamma)\rho-\rho_B\right] \tag{18}$$

$$\frac{B^2{}_4}{B^2}+2\frac{B_4C_4}{BC}-\frac{1}{B^2}-\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi}{\phi}\left[\rho+\rho_B\right] \tag{19}$$

$$\rho_4+(1+w)\rho\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)+\rho\left(\delta\frac{B_4}{B}+\gamma\frac{C_4}{C}\right)+(\rho_B)_4+2\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\rho_B=0 \tag{20}$$

$$\phi_{44}+\phi_4\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{8\pi(\rho-3w\rho+2\rho_B-\gamma\rho)}{(3+2\omega)\phi}. \tag{21}$$

Now we define some physical parameters before solving the field equations. The directional parameters in the direction of x, y and z axes for the Bianchi Type III metric are defined as follows

$$H_x=H_y=\frac{B_4}{B},\ H_z=\frac{C_4}{C}. \tag{22}$$

The mean Hubble parameter is given by

$$H=\frac{1}{3}\frac{V_4}{V}=\frac{1}{3}\left(2\frac{B_4}{B}+\frac{C_4}{C}\right) \tag{23}$$

where $V=a^3=B^2C$ is the volume of the universe. The anisotropy parameter of the expansion is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2. \tag{24}$$

where H_i (i=1, 2, 3) represent the directional Hubble parameters in the direction of x, y and z axes respectively. The equation (24) further reduces to

$$\Delta=\frac{2}{9H^2}\left(H_y-H_z\right)^2 \tag{25}$$

Using equations (17), (18), we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{B_4^2}{B^2}-\frac{B_4C_4}{BC}-\frac{1}{B^2}+\frac{\phi_4}{\phi}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=-\frac{8\pi}{\phi}\left[\gamma\rho+2\rho_B\right]. \tag{26}$$

From equation (26), we obtain

$$H_y-H_z=\frac{B_4}{B}-\frac{C_4}{C}=\frac{\lambda}{V\phi}+\int\left[\frac{-8\pi}{\phi}\left(\gamma\rho+2\rho_B\right)+\frac{1}{B^2}\right]dt, \tag{27}$$

where λ is the real constant of integration and the term with γ is the term that arises due to the possible intrinsic anisotropy of the fluid.

Using equations (25) and (27), we get

$$\Delta=\frac{2}{9H^2}\left\{\frac{\lambda}{V\phi}+\int\left[\frac{-8\pi}{\phi}\left(\gamma\rho+2\rho_B\right)+\frac{1}{B^2}\right]dt\right\}^2. \tag{28}$$

The integral term in equation (28) vanishes for

$$\gamma=\frac{1}{8\pi\rho}\left[\frac{\phi}{B^2}-16\pi\rho_B\right], \tag{29}$$

which also leads to the following energy-momentum tensor

$$T_\nu^\mu = diag\left\{ \begin{array}{l} -w\rho + \rho_B, -\left(w - \frac{2\rho_B}{\rho}\right)\rho - \rho_B, \\ -\left(w + \frac{\phi}{8\pi\rho}\left(\frac{1}{B^2} - \frac{16\pi\rho_B}{\phi}\right)\right) - \rho_B, \rho + \rho_B \end{array} \right\} \tag{30}$$

The anisotropic parameter for the Bianchi type III metric reduces to the following form

$$\Delta = \frac{2\lambda^2}{9\phi^2 H^2} V^{-2}. \tag{31}$$

One can check that this behavior of the Δ in equation (31), we obtained by using an anisotropic fluid (25) in Bianchi type III space-time is equivalent to the ones that can be obtained similarly for Bianchi type I and Bianchi type V space-time by using any isotropic fluid. Then one would see that as $\phi \to 1$, the result we obtain for Δ in the model given below is equivalent to the ones obtained in Kumar *et al.* [33] for Bianchi type I and Singh *et al.* [34,35] for Bianchi type V space-time in case of isotropic fluid.

The vanishing of the integral term also reduces the difference between the expansion rates on *y* and *z* axes to the following form

$$H_y - H_z = \frac{\lambda}{B^2 C\phi}. \tag{32}$$

The power law relation between scalar field ϕ and scale factor '*a*' has already been used by Johri and Desikan [36] in the context of Robertson Walker Brans-Dicke models. Thus, the power law relation between ϕ and *a i.e.* $\phi \propto a^n$ where *n* is any integer implies that

$$\phi = ba^n. \tag{33}$$

where *b* is the constant of proportionality.

The energy conservation equation (12), leads to two equations for the anisotropic fluid and magnetic field [32, 37].

$$\rho_4 + (1+w)\rho\left(2\frac{B_4}{B} + \frac{C_4}{C}\right) + \rho\left(\delta\frac{B_4}{B} + \gamma\frac{C_4}{C}\right) = 0 \tag{34}$$

$$\rho_B = \frac{\alpha}{B^2C^2}. \tag{35}$$

Using equations (29) and (33), the field equations (17)-(19) becomes

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{\phi_{44}}{\phi} = \frac{8\pi}{\phi}\left[-w\rho + \rho_B\right] \tag{36}$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(2\frac{B_4}{B}\right) + \frac{\phi_{44}}{\phi} = \frac{8\pi}{\phi}\left[-w\rho - \frac{\phi}{8\pi}\left(\frac{1}{B^2} - \frac{16\pi\rho_B}{\phi}\right) - \rho_B\right] \tag{37}$$

$$\frac{B_4^2}{B^2} + 2\frac{B_4C_4}{BC} - \frac{1}{B^2} - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(2\frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{8\pi}{\phi}\left[\rho + \rho_B\right] \tag{38}$$

We have three linearly independent equations (36-38) and five unknown functions (B, C, ρ, ρ_B, w). Thus, we can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition; however, these procedures have some drawbacks. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. To solve the above set of highly non-linear equations, we have used two different volumetric expansion laws [31].

$$V = c_1 e^{3mt}, \tag{39}$$

$$V = c_2 t^{3k} \tag{40}$$

where c_1, c_2, k and m are positive constants. In this way, all possible expansion histories, the exponential expansion (39) and the power law expansion (40) have been covered. The models with the exponential expansion and power law for $k > 1$ exhibit accelerating volumetric expansion. On the other hand the model for $k = 1$ exhibits volumetric expansion with constant velocity; the models for $k < 1$ exhibit decelerating volumetric expansion. Thus, phenomenologically, the anisotropic fluid we dealed here can be considered in the context of DE in the models with exponential expansion and power law expansion for $k > 1$.

3. MODEL FOR EXPONENTIAL EXPANSION

After solving the field equation (36)-(38) for the exponential volumetric expansion (39) by considering (15), (32) and (33), we obtain the metric potentials as follows

$$A = B = \left(\frac{c_1}{c_2}\right)^{\frac{1}{3}} \exp\left[mt - \frac{\lambda}{3bm(n+3)c_1^{\frac{n+3}{3}}} e^{-m(n+3)t}\right], \tag{41}$$

$$C=\left(c_1c_2^2\right)^{\frac{1}{3}}\exp\left[mt+\frac{2\lambda}{3bm(n+3)c_1^{\frac{n+3}{3}}}e^{-m(n+3)t}\right] \quad (42)$$

where c_2 is integration constant. For these values of metric potentials the directional Hubble parameters on the *x*, *y* and *z* axes found to be

$$H_x=H_y=m+\frac{\lambda}{3bc_1^{\frac{n+3}{3}}}e^{-m(n+3)t},$$

$$H_z=m-\frac{2\lambda}{3bc_1^{\frac{n+3}{3}}}e^{-m(n+3)t} \quad (43)$$

which are all finite for all finite values of *t*.

The mean Hubble parameter of this model is given by

$$H=m. \quad (44)$$

The deceleration parameter is

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=-1. \quad (45)$$

For any physically relevant model, the Hubble parameter *H* and deceleration parameter *q* are the most important observational quantities in cosmology. Recent observation by Riess *et al.* [38] show that the deceleration parameter of the universe is in the range $-1\le q\le 0$. We observe that the relation (45) gives q as a constant. The sign of q indicates whether the model inflates or not. The negative sign of q correspond to accelerating model whereas the positive sign of q indicates deceleration. In the present case q is negative therefore the universe is accelerating.

Using the directional parameters (43) and mean Hubble parameter (44) in (25), we obtain

$$\Delta=\frac{2\lambda^2}{9m^2b^2c_1^{2[(n+3)/3]}}e^{-2m(n+3)t} \quad (46)$$

we observe that at $t=0$, $\Delta\neq 0$ *i.e.* fluid was anisotropic at early epoch and at $t\to\infty, \Delta\to 0$ *i.e.* at large time fluid isotropizes.

The scalar field of the model is obtained from equation (33), as

$$\phi=bc_1^{n/3}e^{mnt}. \quad (47)$$

The energy density of magnetic field of the model from equations (41), (42) in (35) is found to be

$$\rho_B=\alpha(c_1^2c_2)^{-2/3}\exp\left(-4mt-\frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right). \quad (48)$$

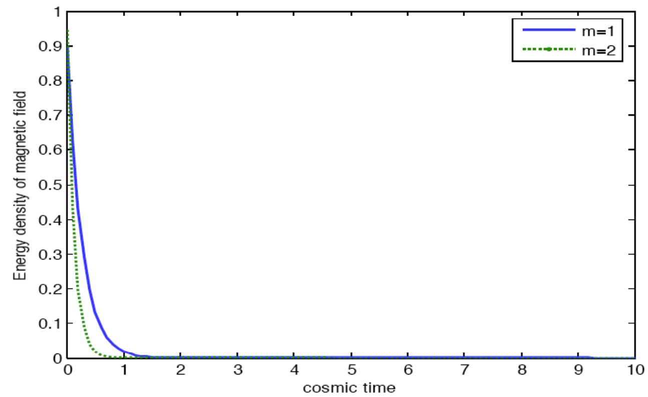

Figure 1: The plot of energy density of magnetic field versus cosmic time with parameters $\alpha=b=c_1=c_2=\lambda=1$ and $n=3$.

From equation (48), it is observed that the energy density of magnetic field is always positive and a decreasing function of time. Figure **1** clearly shows this behavior in accelerating model. The interesting point is that energy density of magnetic field in our model is defined at $t=0$ and we do not have any singularity.

The density of the model from equations (38), (41), (42), (48) is found to be

$$\frac{8\pi\rho}{\phi}=\left\{\begin{array}{l}\frac{-8\pi}{bc_1^{n/3}e^{mnt}}\left[\alpha(c_1^2c_2)^{-2/3}\exp\left(-4mt-\frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right)\right]\\+\left[\frac{6-\omega n^2-6n}{2}\right]m^2-\frac{3\lambda^2e^{-2m(n+3)t}}{9b^2c_1^{2[(n+3)/3]}}\\-\left(\frac{c_1}{c_2}\right)^{-2/3}\exp\left(-2mt+\frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right)\end{array}\right\}. \quad (49)$$

The energy density of universe with cosmic time is clearly shown in Figure **2**. We conclude that, it is evolving with decreasing function of time in accelerating phase of universe and in early stage, the energy density was zero, due to coupling of magnetic field and dark energy and it tends to negative.

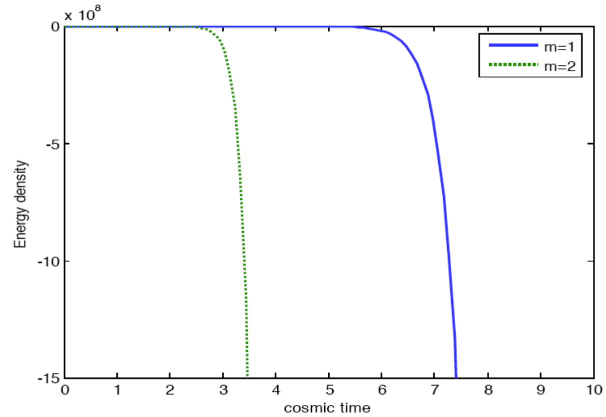

Figure 2: The plot of energy density versus cosmic time with parameters $\alpha = b = c_1 = c_2 = \lambda = 1$ $\omega = 0.5$ and $n = 3$.

The skewness parameter is obtained by using (33), (41), (49) in (29) as

$$\frac{8\pi\gamma\rho}{\phi} = \left\{ \begin{array}{l} \left[\left(\frac{c_1}{c_2}\right)^{-2/3} \exp\left(-2mt + \frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right)\right. \\ \left. -\frac{16\pi}{bc_1^{n/3}e^{mnt}}\left[\alpha(c_1^2c_2)^{-2/3}\exp\left(-4mt - \frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right)\right]\right] \end{array}\right\} \tag{50}$$

Using (36), (41), (42) and (49), we obtain

$$\frac{8\pi w\rho}{\phi} = \left\{ \begin{array}{l} \frac{8\pi}{bc_1^{n/3}e^{mnt}}\left[\alpha(c_1^2c_2)^{-2/3}\exp\left(-4mt - \frac{2\lambda e^{-m(n+3)t}}{3bm(n+3)c_1^{(n+3)/3}}\right)\right] \\ -\left[\frac{6+\omega n^2+4n+2n^2}{2}\right]m^2 - \frac{3\lambda^2 e^{-2m(n+3)t}}{9b^2c_1^{2[(n+3)/3]}} \end{array}\right\} \tag{51}$$

The variation of equation of state parameter with cosmic time is shown in Figure **3**, as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well known situations. We see that in early stage, the EoS parameter was zero *i.e.* $w = 0$ (dusty universe dominated) and at late time it is evolving with positive value (the universe is matter dominated).

In the absence of magnetic field *i.e.* $\alpha \to 0$, the energy density of magnetic field, the energy density of fluid, the skewness parameter and deviation free parameter of the model are similar to the result obtained by Shamir *et al.* [30].

In General Relativity, (For n=0, $\omega \to \infty$, $\phi \to 1$, b=1) when $\alpha \to 0$ (In absence of magnetic field) the energy density, Skewness Parameter and deviation free Parameter results resembles to that of Akarsu *et al.* [31].

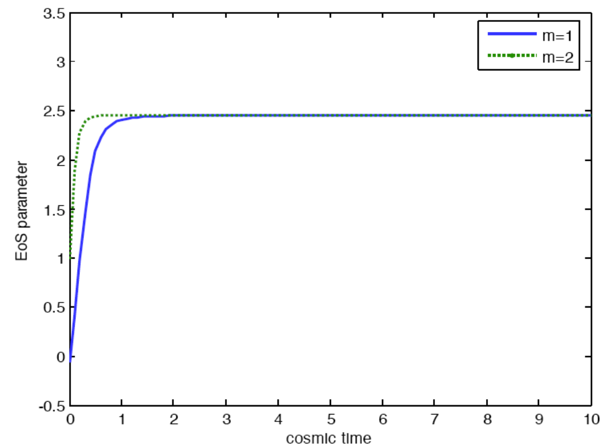

Figure 3: The plot of EoS Parameter versus cosmic time with $b = c_1 = c_2 = \alpha = \lambda = 1$, $n = 3$ and $\omega = 0.5$.

4. MODEL FOR POWER LAW EXPANSION

After solving the field equations (36)-(38) for the power law expansion (40) by considering (15), (32) and (33), we obtain the scale factors as follows

$$A = B = \left(\frac{c_2}{c_3}\right)^{\frac{1}{3}} t^k \exp\left[\frac{\lambda}{3b[1-k(n+3)]k^{\frac{n+3}{3}}} t^{1-k(n+3)}\right], \tag{52}$$

$$C = \left(c_2 c_3^2\right)^{\frac{1}{3}} t^k \exp\left[\frac{-2\lambda}{3b[1-k(n+3)]k^{\frac{n+3}{3}}} t^{1-k(n+3)}\right] \tag{53}$$

where c_3 is constant of integration. For these solutions the directional Hubble parameters on the *x*, *y* and *z* axes are

$$H_x = H_y = \frac{k}{t} + \frac{\lambda}{3bk^{(n+3)/3}} t^{-k(n+3)}, \; H_z = \frac{k}{t} - \frac{2\lambda}{3bk^{(n+3)/3}} t^{-k(n+3)}. \tag{54}$$

The mean Hubble parameter of this model is given by

$$H = \frac{k}{t}. \tag{55}$$

It is clear that expansion rate of universe decrease with time.

The deceleration parameter is obtained as

$$q = \frac{1-k}{k} \tag{56}$$

We observe that the model is accelerating for the value of $k > 0$.

From equations (54), (55) in (25), we get

$$\Delta = \frac{2\lambda^2}{9bk^2 c_1^{2[(n+3)/3]}} t^{2-6k}. \quad (57)$$

From equation (57), it is clear that at $t = 0, \Delta \neq 0$ *i.e.* fluid was anisotropic at early stage of evolution and at $t \to \infty, \Delta \to \infty$ therefore fluid isotropizes at large time.

The energy density of magnetic field of the model from equations (52) and (53) in (35) is found to be

$$\rho_B = \left[\alpha\left(c_2^2 c_3\right)^{-2/3} t^{-4k} \exp\left(\frac{2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right)\right], \quad (58)$$

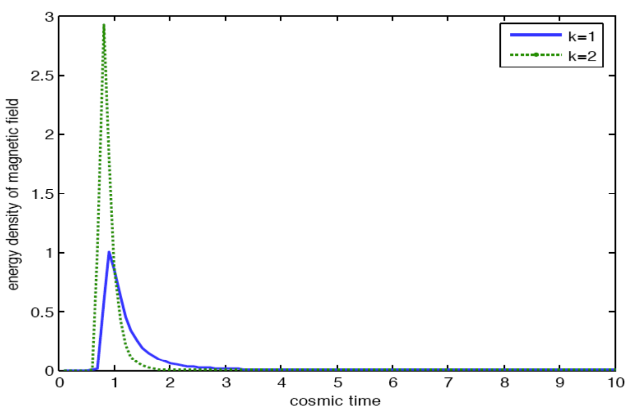

Figure 4: The plot of energy density of magnetic field versus cosmic time with $b = c_2 = c_3 = \alpha = \lambda = 1$ and $n = 3$.

From Figure **4**, at early epoch, the energy density of magnetic field was zero and it increases rapidly within a short period to its maximum value and thereafter decreases with cosmic time in both inflationary $(k = 1)$ and accelerating $(k = 2)$ models [39].

The density of the model from equations (38), (47), (52), (53) and (58) is obtained as

$$\frac{8\pi\rho}{\phi} = \left\{\begin{array}{l} \frac{-8\pi}{bk^{n/3}t^{nk}}\left[\alpha\left(c_2^2 c_3\right)^{-2/3} t^{-4k} \exp\left(\frac{2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right)\right] \\ +\left(\frac{6-\omega n^2+6n}{2}\right)\frac{k^2}{t^2} - \frac{3\lambda^2 t^{-2k(n+3)}}{9b^2 c_2^{2[(n+3)/3]}} \\ -\left(\frac{c_2}{c_3}\right)^{-2/3} t^{-2k} \exp\left(\frac{-2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right) \end{array}\right\} . \quad (59)$$

The skewness parameter is obtained by using (33), (52), (53), (58) and (59) in (29) as

$$\frac{8\pi\gamma\rho}{\phi} = \left\{\begin{array}{l} \left(\frac{c_2}{c_3}\right)^{-2/3} t^{-2k} \exp\left(\frac{2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right) \\ -\left[\frac{16\pi\alpha}{bc_2^{n/3}t^{nk}}\left(c_2^2 c_3\right)^{-2/3} t^{-4k} \exp\left(\frac{2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right)\right] \end{array}\right\} \quad (60)$$

Using (36), (47), (52), (53), (58) and (59), we obtain

$$\frac{8\pi w\rho}{\phi} = \left\{\begin{array}{l} \frac{8\pi\alpha}{bc_2^{n/3}t^{nk}}\left(c_2^2 c_3\right)^{-2/3} t^{-4k} \exp\left(\frac{2\lambda t^{1-k(n+3)}}{3b[1-k(n+3)]c_2^{(n+3)/3}}\right) \\ -\left(\frac{6+\omega n^2+4n+2n^2}{2}\right)\frac{k^2}{t^2} + (n+2)\frac{k}{t^2} - \frac{3\lambda^2 t^{-2k(n+3)}}{9b^2 c_2^{2[(n+3)/3]}} \end{array}\right\} \quad (61)$$

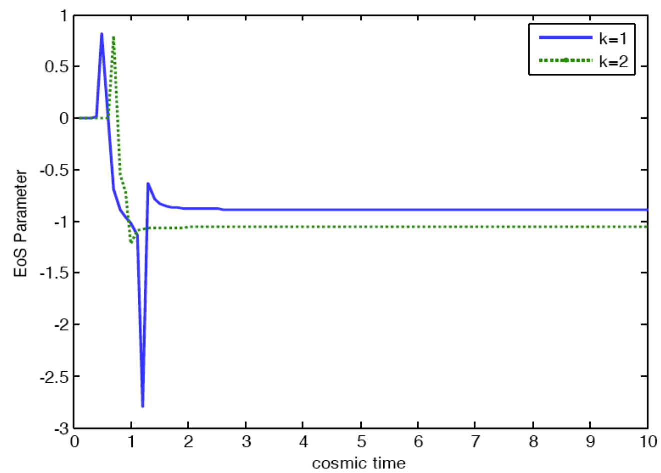

Figure 5: The plot of EoS parameter versus cosmic time with, $\omega = 0.5$, $n = 3$ and $b = c_2 = c_3 = \alpha = \lambda = 1$.

The variation of equation of state parameter (w) with cosmic time (t) is shown in Figure **5**, as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well known situations. We see that in early stage, the EoS parameter was zero (dusty universe dominated) and at late time it is evolving with negative value (at the present time). The earlier real matter later on converted to the dark energy dominated phase of universe in accelerating models ([12], [40]). Therefore, it follows that our dark energy model in Brans-Dicke Theory is consistent with the recent observation of Type Ia Supernovae [38].

In the absence of magnetic field *i.e.* $\alpha \to 0$, the energy density of fluid, the skewness parameter and deviation free parameter results are resembles with Shamir *et al.* [30].

In general Relativity (for n=0, $\omega \to \infty$, $\phi \to 1$, b=1) when $\alpha \to 0$ (In absence of Magnetic field), the energy density, Skewness parameter and deviation free parameter results are similar to Akarsu *et al.* [31].

5. CONCLUDING REMARKS

In this paper, we have presented Bianchi Type III cosmological models with magnetized anisotropic dark energy in scalar tensor theory of gravitation proposed by Brans-Dicke [1]. The exact solutions of the Brans-Dicke field equations have been obtained by assuming two different volumetric expansion laws in a way to cover all possible expansions namely exponential expansion and power law expansion.

The main features of the model are as

(i) In exponential expansion law, it is observed that energy density of magnetic field is always positive and a decreasing function of time for accelerating model. In early stage the EoS parameter (w) was zero (dusty dominated universe) and at late time it is evolving with positive value (the universe is matter dominated). It is also found that the energy density of universe is decreasing function of time. At early stage energy density was zero and which tends to negative at large time due to coupling of magnetic field and dark energy.

(ii) In power law expansion, it is observed that at early epoch the energy density of magnetic field was zero and it increases rapidly within a short period of time to its maximum value and there after decreases with time [39] for both inflationary and accelerating models. In early stage the EoS parameter (w) was zero (dusty universe) and at late time it is evolving with negative value (at the present time). Thus, our DE model represents realistic model [12, 40].

(iii) Some important cosmological physical parameters for the solutions such as Hubble parameter, mean Hubble parameter, Anisotropy parameter, Energy density of fluid, deviation free parameters are examined and discussed in both models

(iv) From the relation (45) it is clear that our universe is accelerating. It is resemble to the current observations of SNe Ia and CMBR [41]. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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